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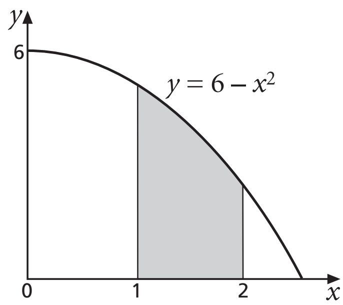
6 Integration 2

In this chapter you will learn how to find

- the area under a curve defined parametrically
- the volume of a solid of revolution about the *x*-axis
- the percentage error when the trapezium rule is used to estimate the value of a definite integral

A Area under a curve defined parametrically (answers p 177)

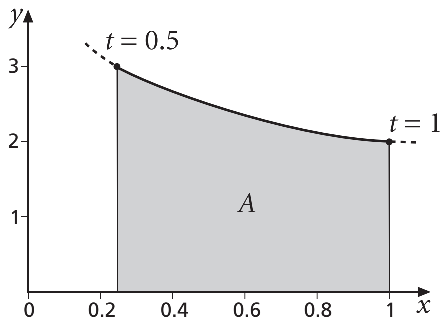


The diagram shows the area under the curve whose equation is $y = 6 - x^2$, between the points where $x = 1$ and $x = 2$.

To find this area you calculate $\int_1^2 (6 - x^2)\,dx$.

The general form of this integral can be written $\int_a^b y\,dx$, where *a* and *b* are the values of *x* at the end-points.

In the example above, the equation of the curve is given in the explicit form $y =$ a function of x.



In the example on the right, the curve is defined by the parametric equations

$$x = t^2, \qquad y = 1 + \frac{1}{t}$$

The shaded area *A* is between the points where $t = 0.5$ and $t = 1$.

The integral $\int_a^b y\,dx$, which gives the area, has to be rewritten in terms of *t*.

This is like integration by substitution: *y* is replaced by $1 + \frac{1}{t}$ and d*x* is replaced by $\frac{dx}{dt}\,dt$, or $2t\,dt$.

The lower and upper limits are the two values of *t* between which the area lies.

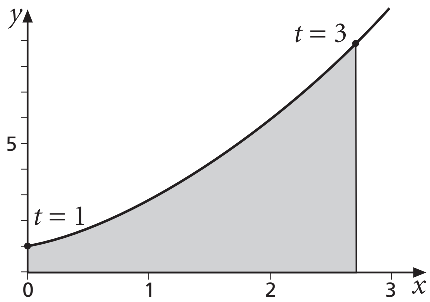
So the area *A* is $\int_{0.5}^{1} \left(1 + \frac{1}{t}\right) 2t\,dt$.

A1 Evaluate the integral above to find the shaded area *A*.

A2 The diagram shows part of the curve defined by the parametric equations

$$x = t - \frac{1}{t}, \quad y = t^2$$

Find the area under the curve between the points $t = 1$ and $t = 3$.

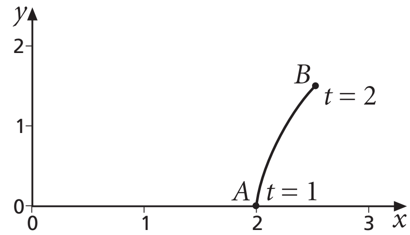


A3 *AB* is part of the curve whose parametric equations are

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}$$

A is the point where $t = 1$. *B* is the point where $t = 2$.

Find the area under the graph between *A* and *B*.

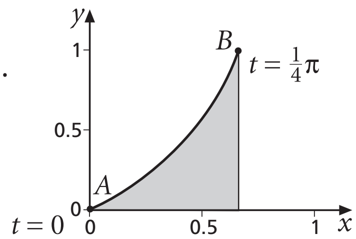


K The area under a curve defined by parametric equations is given by $\int_{t_1}^{t_2} y\,\mathrm{d}x$ where y and dx are each expressed in terms of the parameter t, and t_1 and t_2 are the values of t at the end-points of the curve.

Example 1

The diagram shows part of the graph of the function defined by the parametric equations $x = \sin t,\ y = \tan t$. *A* is the point where $t = 0$, *B* the point where $t = \frac{1}{4}\pi$.

Find the area under the graph between *A* and *B*.



Solution

The area is given by $\int y\,\mathrm{d}x$.

$y = \tan t, \quad \frac{\mathrm{d}x}{\mathrm{d}t} = \cos t, \quad$ so $\mathrm{d}x = \cos t\,\mathrm{d}t$.

$$\text{Area} = \int_0^{\frac{1}{4}\pi} \tan t \cos t\,\mathrm{d}t = \int_0^{\frac{1}{4}\pi} \frac{\sin t}{\cos t}\cos t\,\mathrm{d}t = \int_0^{\frac{1}{4}\pi} \sin t\,\mathrm{d}t = \left[-\cos t\right]_0^{\frac{1}{4}\pi} = \left(-\frac{1}{\sqrt{2}}\right) - (-1) = 1 - \frac{1}{\sqrt{2}}$$

Exercise A (answers p 177)

1 Find the area under the curve whose parametric equations are $x = t^2,\ y = t + \frac{1}{t}$ between the points $t = \frac{1}{2}$ and $t = 2$.

2 In each case below, find the area under the curve whose parametric equations are given, between the points whose values of t are given (in square brackets).

(a) $x = 2(t^2 - 1),\ y = 3(t + 1)\ \ [1,\ 3]$

(b) $x = 1 + \sqrt{t},\ y = t(1 + t)\ \ [0,\ 1]$

(c) $x = t + \ln t,\ y = t - \frac{1}{t}\ \ [1,\ 2]$

(d) $x = \sin t,\ y = 1 - \sec t\ \ [0,\ \frac{1}{6}\pi]$

(e) $x = \cos 2t,\ y = \sec t\ \ [0,\ \frac{1}{4}\pi]$

(f) $x = t^2,\ y = \mathrm{e}^t\ \ [0,\ 2]$

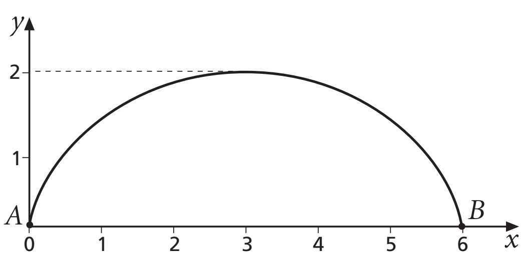
3 The curve shown here is part of the ellipse whose parametric equations are

$$x = 3(1 - \cos t), \quad y = 2\sin t$$

(a) What is the value of t at the point

(i) *A* **(ii)** *B*

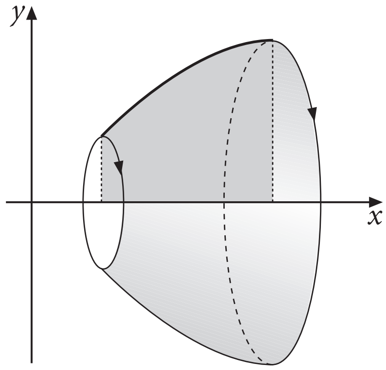
(b) Find the area enclosed by the curve and the x-axis.



B Volume of a solid revolution about the *x*-axis (answers p 177)

In this diagram the shaded region under a curve is rotated through a complete turn (2π radians) about the x-axis to form a **solid of revolution**.

In this section you will learn how to calculate the volume of a solid of revolution when you know the equation of the curve.



The second diagram shows the volume V up to the value x. The radius of the solid at this value is y.

If x is increased by an amount δx, then y is increased by δy and V is increased by δV.

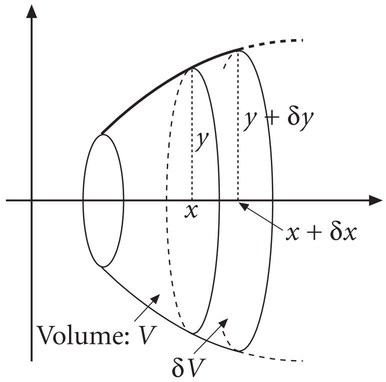
The extra volume δV consists of a slice of thickness δx, whose cross-sectional area varies from a circle of radius y to a circle of radius $(y + \delta y)$.

The volume of the slice lies between $\pi y^2 \delta x$ and $\pi(y + \delta y)^2 \delta x$.

$$\pi y^2 \delta x < \delta V < \pi(y + \delta y)^2 \delta x$$

By dividing by δx, we get

$$\pi y^2 < \frac{\delta V}{\delta x} < \pi(y + \delta y)^2$$



Now suppose that δx gets smaller and smaller.

Then δy gets smaller and smaller and $\frac{\delta V}{\delta x}$ gets closer and closer to $\frac{dV}{dx}$, which is 'trapped' between πy^2 and a quantity that gets closer and closer to πy^2.

So $\frac{dV}{dx} = \pi y^2$ and this is equivalent to $V = \int \pi y^2 \, dx$.

To find the volume between two given values of x, you calculate a definite integral:

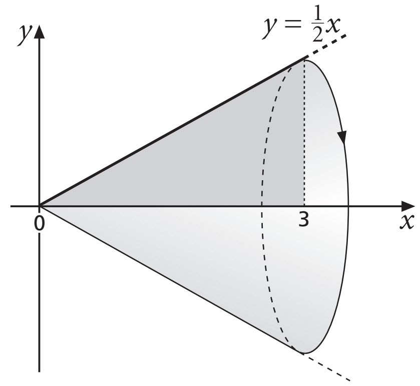
K The volume of a solid of revolution about the x-axis between $x = a$ and $x = b$ is given by $\int_a^b \pi y^2 \, dx$.

B1 A solid of revolution is formed by rotating about the x-axis the region under the line $y = \frac{1}{2}x$ between $x = 0$ and $x = 3$.

(a) What name is given to this solid of revolution?

(b) Express y^2 in terms of x.

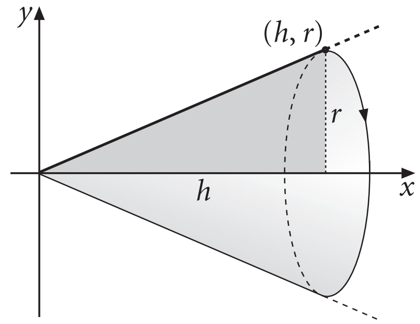
(c) By substituting for y^2 in $\int_0^3 \pi y^2 \, dx$, find the volume of the solid of revolution.



B2 The sloping line in this diagram goes through the point (h, r). When the shaded area is rotated through 2π radians about the x-axis, the solid of revolution formed is a cone of base radius r and height h.

(a) Explain why the equation of the line is $y = \frac{r}{h}x$.

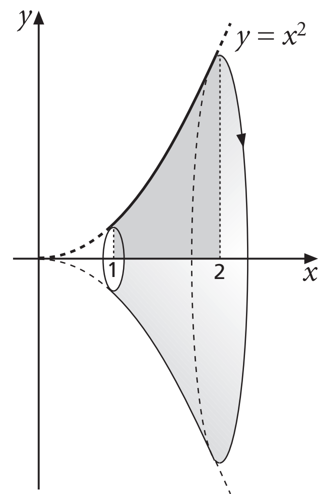
(b) By substituting for y^2 in $\int_0^h \pi y^2 \, dx$, show that the volume of the cone is $\frac{1}{3}\pi r^2 h$.



B3 A solid of revolution is formed by rotating about the x-axis the region under the curve $y = x^2$ between $x = 1$ and $x = 2$.

(a) Express y^2 in terms of x.

(b) Find the volume of the solid of revolution.

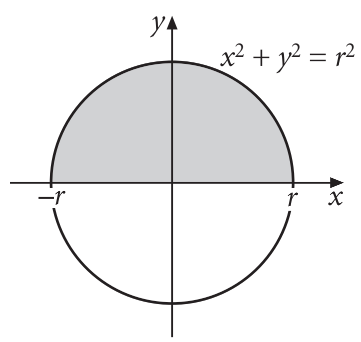


B4 This diagram shows the circle of radius r whose centre is at the origin. The equation of the circle is $x^2 + y^2 = r^2$, which can be written as $y^2 = r^2 - x^2$.

(a) What solid is formed when the shaded region is rotated through 2π radians about the x-axis?

(b) The volume of this solid is given by $\int_{-r}^{r} \pi y^2 \, dx$.

Using the fact that $y^2 = r^2 - x^2$, show that the volume of the solid of revolution is $\frac{4}{3}\pi r^3$.



The integration that you need to do in order to find the volume of a solid of revolution may require one of the techniques you met in the previous chapter.

Example 2

The region under the curve $y = \sqrt{\frac{1+x}{x}}$ between $x = 1$ and $x = 4$ is rotated through 2π radians about the x-axis to form a solid of revolution. Find the volume of the solid.

Solution

$$\int_1^4 \pi y^2\,dx = \int_1^4 \pi\left(\frac{1+x}{x}\right)dx = \pi\int_1^4\left(\frac{1}{x}+\frac{x}{x}\right)dx = \pi\int_1^4\left(\frac{1}{x}+1\right)dx$$

$$= \pi\left[\ln|x| + x\right]_1^4 = \pi\left((\ln 4 + 4) - (0+1)\right) = \pi(\ln 4 + 3)$$

Example 3

The region under the curve $y = \sqrt{\frac{2x}{x^2+3}}$ between $x = 0$ and $x = 1$ is rotated through 2π radians about the x-axis to form a solid of revolution. Find the volume of the solid.

Solution

$$\text{Volume} = \int_0^1 \pi y^2\,dx = \pi\int_0^1 \frac{2x}{x^2+3}\,dx \qquad \left(\textit{This integral is of the form } \int \frac{f'(x)}{f(x)}\,dx.\right)$$

$$= \pi\left[\ln\left|x^2+3\right|\right]_0^1 = \pi(\ln 4 - \ln 3) = \pi \ln\tfrac{4}{3}$$

Exercise B (answers p 177)

1 The region under the curve $y = \sqrt{x(4-x)}$ between $x = 0$ and $x = 4$ is rotated through 2π radians about the x-axis.
Find the volume of the solid of revolution formed.

2 The region under the curve $y = 1 - x^2$ between $x = 0$ and $x = 1$ is rotated through 2π radians about the x-axis.

Find the volume of the solid of revolution formed.

3 Find the volume of the solid of revolution formed when the region enclosed by each of these curves and the x-axis, between the given values of x, is rotated through 2π radians about the x-axis.

(a) $y = e^x$ from $x = 0$ to $x = 3$

(b) $y = \frac{1}{x^2}$ from $x = 1$ to $x = 4$

(c) $y = x + \frac{1}{x}$ from $x = 1$ to $x = 3$

(d) $y = \sqrt{\sin 3x}$ from $x = 0$ to $x = \frac{1}{3}\pi$

(e) $y = \sqrt{x \sin 2x}$ from $x = 0$ to $x = \frac{1}{2}\pi$

(f) $y = \sqrt{xe^{-x}}$ from $x = 0$ to $x = 2$

(g) $y = \frac{1}{\sqrt{1+2x}}$ from $x = 0$ to $x = 1$

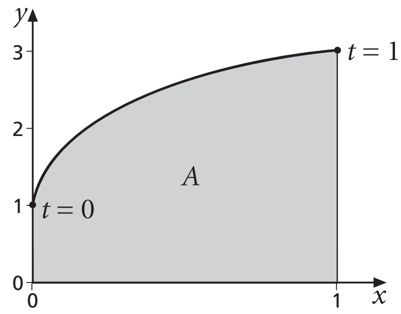
(h) $y = \sqrt{\frac{x}{x^2+1}}$ from $x = 0$ to $x = 3$

C Solid of revolution defined parametrically (answers p 178)

If the curve that is rotated is defined parametrically, the volume is still $\int \pi y^2\,dx$, but you need to substitute for y, for dx and for the limits of integration.

For example, here is the part of the curve whose parametric equations are

$$x = t^2, \qquad y = 1 + 2t$$



The shaded area A between the points where $t = 0$ and $t = 1$ is rotated through 2π radians about the x-axis.

The volume is given by $\int_0^1 \pi y^2\,dx$.

$y^2 = (1 + 2t)^2$, $dx = 2t\,dt$, so $\int_0^1 \pi y^2\,dx = \pi\int_0^1 (1+2t)^2\,2t\,dt$

C1 Show that the value of the integral above is $\dfrac{17\pi}{3}$.

Example 4

The region between $t = 0$ and $t = \frac{1}{4}\pi$ under the curve with parametric equations $x = \tan t$, $y = \sin t$ is rotated through 2π radians about the x-axis.

Find the volume of the solid of revolution formed.

Solution

$y^2 = \sin^2 t$, $\quad dx = \sec^2 t\,dt$

$$\text{Volume} = \int_0^{\frac{1}{4}\pi} \pi y^2\,dx = \pi\int_0^{\frac{1}{4}\pi} \sin^2 t\sec^2 t\,dt$$

$$= \pi\int_0^{\frac{1}{4}\pi} \frac{\sin^2 t}{\cos^2 t}\,dt = \pi\int_0^{\frac{1}{4}\pi} \tan^2 t\,dt = \pi\int_0^{\frac{1}{4}\pi} \left(\sec^2 t - 1\right)dt$$

$$= \pi\left[\tan t - t\right]_0^{\frac{1}{4}\pi} = \pi\left(1 - \tfrac{1}{4}\pi\right)$$

Exercise C (answers p 178)

1 Find the volume of the solid of revolution formed when the region between $t = 0$ and $t = 2$ under the curve with parametric equations $x = 2t^2$, $y = 3(t - 1)$ is rotated through 2π radians about the x-axis.

2 In each case below, find the volume of the solid of revolution formed when the region under the curve with the given parametric equations, between the given values of the parameter (in square brackets), is rotated through 2π radians about the x-axis.

(a) $x = t^2 + 1$, $y = t^3 - 1$ $[0,\ 1]$ **(b)** $x = t + \ln t$, $y = t^2$ $[1,\ e]$

(c) $x = \sqrt{t} + t$, $y = \sqrt{t} - t$ $[0,\ 1]$ **(d)** $x = \sin\theta$, $y = \sqrt{\sin\theta + \cos\theta}$ $[0,\ \frac{1}{2}\pi]$

3 Find the volume of the solid of revolution formed when the region between the points $t = 1$ and $t = 2$ under the curve with parametric equations $x = t + \dfrac{1}{\sqrt{t}}$, $y = t - \dfrac{1}{\sqrt{t}}$ is rotated through 2π radians about the x-axis.

4 The region between the points $t = 0$ and $t = 2$ under the curve with parametric equations $x = t^2$, $y = e^t$ is rotated through 2π radians about the x-axis.
Find the volume of the solid of revolution formed.

5 Find the volume of the solid of revolution formed when the region between $\theta = \frac{1}{4}\pi$ and $\theta = \frac{1}{3}\pi$ under the curve with parametric equations $x = \ln\sin\theta$, $y = \tan\theta$ is rotated through 2π radians about the x-axis.

6 **(a)** Show that the derivative of $\ln(\sec x + \tan x)$ is $\sec x$.

(b) The region between $\theta = 0$ and $\theta = \frac{1}{4}\pi$ under the curve with parametric equations $x = \sin\theta$, $y = \tan\theta$ is rotated through 2π radians about the x-axis. Show that the volume of the solid of revolution formed is given by

$$\pi\int_0^{\frac{1}{4}\pi}(\sec\theta - \cos\theta)\,d\theta$$

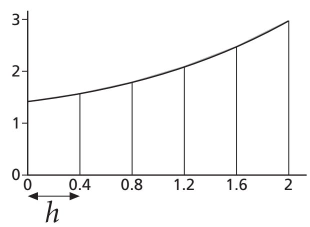
(c) Show that the value of this integral is $\pi\left[\ln\left(1+\sqrt{2}\right) - \frac{1}{2}\sqrt{2}\right]$.

D Trapezium rule: percentage error (answers p 178)

The function $\sqrt{1+e^x}$ cannot be integrated by the methods you have met so far.

However, the trapezium rule can be used to estimate a definite integral such as $\int_0^2 \sqrt{1+e^x}\,dx$.

The trapezium rule is obtained by replacing the graph of the function by a set of straight line segments, as shown in the diagram.



The total area of the trapezia formed in this way is an estimate of the area under the graph.

Estimate = $\frac{1}{2}h\left[\text{sum of end ordinates} + 2(\text{sum of other ordinates})\right]$

D1 **(a)** Copy and complete this table of values for the function $y = \sqrt{1+e^x}$.

x	0	0.4	0.8	1.2	1.6	2
y	1.414	1.579				

(b) Use the trapezium rule with six ordinates to find an estimate of $\int_0^2 \sqrt{1+e^x}\,dx$.

(c) How can you tell from the graph that the result in (b) is an overestimate of the value of the integral?

(d) How could you improve on the estimate?

If the trapezium rule is used for a function that can be integrated, the exact value of the integral can be found and hence the percentage error in the estimate.

Example 5

The region under the curve $y = e^{-x}$ between $x = 0$ and $x = 1$ is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Use the trapezium rule with three ordinates to estimate the volume of the solid.

(b) Find by integration the exact value of the volume.

(c) Find the percentage error in the estimate in (a).

Solution

The volume is given by $\int_0^1 \pi y^2\,dx = \pi\int_0^1 e^{-2x}\,dx$

x	0	0.5	1
e^{-2x}	1	0.3679	0.1353

(a) Three ordinates means two strips, so $h = 0.5$. Values of e^{-2x} are:

Estimate $= \pi\times 0.25\times(1 + 0.1353 + 2\times 0.3679) = 1.47$ (to 3 s.f.)

(b) Volume $= \pi\int_0^1 e^{-2x}\,dx = \pi\left[-\tfrac{1}{2}e^{-2x}\right]_0^1 = -\tfrac{1}{2}\pi\left(e^{-2} - 1\right) = \tfrac{1}{2}\pi\left(1 - e^{-2}\right)$

(c) Numerical value of volume $= \tfrac{1}{2}\pi(1 - e^{-2}) = 1.36$ (to 3 s.f.)

Actual error $= 1.469\ldots - 1.358\ldots = 0.111\ldots$ Percentage error $= \frac{0.111\ldots}{1.358\ldots}\times 100 = 8.2\%$ (to 1 d.p.).

Exercise D (answers p 178)

1 Use the trapezium rule with six ordinates to estimate the value of $\int_1^5 \sqrt{\ln x}\,dx$.

2 **(a)** Use the trapezium rule with three ordinates to estimate the value of $\int_0^2 \sqrt{4 - x^2}\,dx$.

(b) Obtain a better estimate by using five ordinates.

(c) Use the substitution $x = 2\sin\theta$ to find the exact value of the integral.

(d) Find the percentage error in the estimate obtained by using the trapezium rule with

(i) 3 ordinates **(ii)** 5 ordinates

Comment on the results.

3 **(a)** Use the trapezium rule with three ordinates to estimate the value of $\int_1^3 x\ln x\,dx$.

(b) Obtain a better estimate by using five ordinates.

(c) Use integration by parts to show that the exact value of the integral is $\frac{9}{2}\ln 3 - 2$.

(d) Find the percentage error in the estimate obtained by using the trapezium rule with

(i) 3 ordinates **(ii)** 5 ordinates

Comment on the results.

4 The region under the graph of $y = \ln x$ between $x = 1$ and $x = 3$ is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Use the trapezium rule with five ordinates to estimate the volume of the solid.

(b) By using the substitution $x = e^u$, find the exact value of the volume.

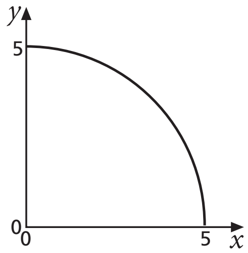
(c) Find the percentage error in the estimate of the volume.

Key points

- The area under a curve defined by parametric equations is given by $\int_{t_1}^{t_2} y\,dx$ where y and dx are each expressed in terms of the parameter t, and t_1 and t_2 are the values of t at the end-points of the curve. (p 79)
- The volume of a solid of revolution about the x-axis between $x = a$ and $x = b$ is given by $\int_a^b \pi y^2\,dx$.

  If x and y are defined in terms of a parameter t, then the volume is $\int_{t_1}^{t_2} \pi y^2\,dx$, where y and dx are each expressed in terms of the parameter t. (pp 80, 83)
- The trapezium rule gives an estimate of the value of a definite integral: $\frac{1}{2}h\left[\text{sum of end ordinates} + 2(\text{sum of other ordinates})\right]$, where h is the gap between ordinates. (p 84)

Mixed questions (answers p 179)



1 The parametric equations for the quarter-circle of radius 5 between (0, 5) and (5, 0) may be written as

$$x = 5\sin\theta,\ y = 5\cos\theta \quad \left(0 \le \theta \le \tfrac{1}{2}\pi\right)$$

Show by integration that the area of the quarter-circle is $\frac{25\pi}{4}$.

2 The region R bounded by the curve $y = x^{\frac{1}{2}}e^{3x}$, the x-axis and the lines $x = 1$ and $x = 4$ is rotated through 2π radians about the x-axis. Find the volume of the solid of revolution formed.

3 The region between $x = 0$ and $x = 3$ under the curve $y = \frac{1}{\sqrt{9 + x^2}}$ is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Use the trapezium rule with four ordinates to estimate the volume of the solid.

(b) By using the substitution $x = 3\tan\theta$, show that the exact value of the volume is $\frac{1}{12}\pi^2$.

(c) Find the percentage error in the estimated volume.

4 A curve has the parametric equations $x = \theta + \tan\theta$, $y = \cos\theta$.
The region under the curve between $\theta = 0$ and $\theta = \frac{1}{4}\pi$ is rotated through 2π radians about the x-axis to form a solid of revolution.
Show that the volume of the solid is $\frac{\pi}{4} + \frac{3\pi^2}{8}$.

5 **(a)** Use integration by parts to show that $\int_0^1 x^2e^{-x}\,dx = 2 - 5e^{-1}$.

(b) Find the volume of the solid of revolution formed when the region under the curve $y = (1 + x)e^{-\frac{1}{2}x}$ between $x = 0$ and $x = 1$ is rotated through 2π radians about the x-axis.

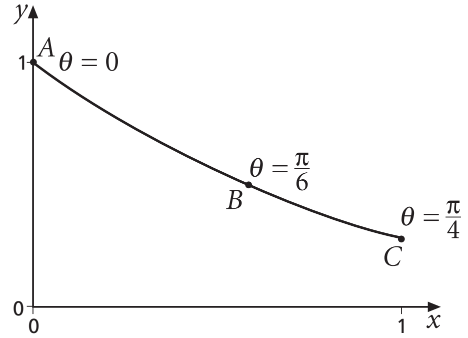
6 ABC is part of the curve whose parametric equations are $x = \tan\theta$, $y = 1 - \sin\theta$.

A is the point where $\theta = 0$, B is the point where $\theta = \frac{1}{6}\pi$ and C is the point where $\theta = \frac{1}{4}\pi$.

(a) Estimate the area under the curve by replacing the curve ABC by two line segments AB and BC.

(b) Find the exact value of the area.

(c) Find the percentage error in the estimate.

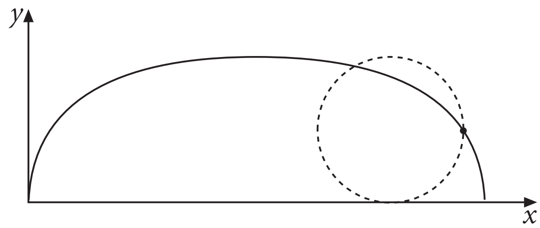


Test yourself (answers p 179)

1 When a wheel of unit radius rolls along horizontal ground, the locus of a point on its circumference is a curve called a 'cycloid'.

The parametric equations of this curve can be written

$x = \theta - \sin\theta$, $y = 1 - \cos\theta$



Find the area under the curve between the points where $\theta = 0$ and $\theta = 2\pi$.

2 The region between the points $t = 0$ and $t = 1$ under the curve whose parametric equations are $x = \ln t$, $y = te^t$ is rotated through 2π radians about the x-axis. Find the volume of the solid of revolution generated.

3 **(a)** Use the trapezium rule with five ordinates to estimate the area under the curve $y = \dfrac{1}{1+\sqrt{x}}$ between $x = 0$ and $x = 1$.

(b) Find the exact value of the area by integration, using the substitution $u = 1 + \sqrt{x}$ or otherwise.

(c) Find the percentage error in the estimate in part (a).

4 **(a)** Use integration by parts to show that

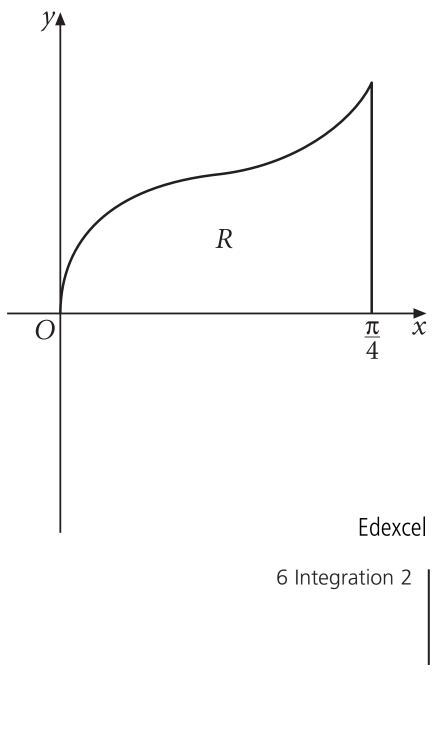
$$\int_0^{\frac{\pi}{4}} x\sec^2 x\,dx = \tfrac{1}{4}\pi - \tfrac{1}{2}\ln 2$$

The finite region R, bounded by the curve with equation $y = x^{\frac{1}{2}}\sec x$, the line $x = \dfrac{\pi}{4}$ and the x-axis, is shown in the diagram.

The region R is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid of revolution generated.

(c) Find the gradient of the curve with equation $y = x^{\frac{1}{2}}\sec x$ at the point where $x = \dfrac{\pi}{4}$.



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