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5 Trigonometry 1

In this chapter you will

- revise the sine and cosine ratios between 0° and 180°
- find lengths and angles in any triangle, using the sine and cosine rules
- calculate the area of any triangle
- learn about radian measure for angles
- calculate lengths of arcs and areas of sectors of circles

A Sine and cosine: revision (answers p 137)

The diagram shows a circle with radius 1 unit.

This is called the unit circle.

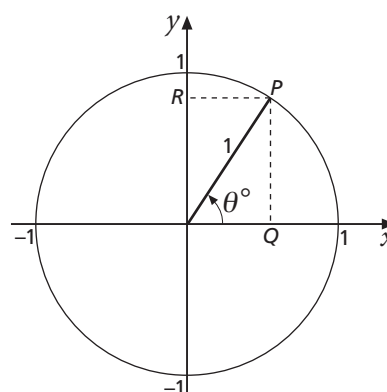
The angle θ° is measured in an anticlockwise direction from the positive x -axis.

A1 Explain why the coordinates of P are $(\cos \theta^\circ, \sin \theta^\circ)$.

$\cos \theta^\circ$ can be defined as the x -coordinate of P .

$\sin \theta^\circ$ can be defined as the y -coordinate of P .

This allows us to define $\cos \theta^\circ$ and $\sin \theta^\circ$ for any angle θ° .



A2 Without using a calculator, write down each of these.

- | | | | |
|----------------------|----------------------|----------------------|----------------------|
| (a) $\cos 0^\circ$ | (b) $\sin 0^\circ$ | (c) $\sin 90^\circ$ | (d) $\cos 90^\circ$ |
| (e) $\sin 180^\circ$ | (f) $\cos 270^\circ$ | (g) $\sin -90^\circ$ | (h) $\cos -90^\circ$ |

A3 Without using a calculator, write down whether each of these is positive or negative.

- | | | | |
|----------------------|----------------------|----------------------|-----------------------|
| (a) $\cos 30^\circ$ | (b) $\sin 120^\circ$ | (c) $\cos 120^\circ$ | (d) $\cos 200^\circ$ |
| (e) $\sin 330^\circ$ | (f) $\cos 330^\circ$ | (g) $\sin 220^\circ$ | (h) $\cos -120^\circ$ |

A4 (a) When $90^\circ < \theta^\circ < 180^\circ$, is the x -coordinate of P positive or negative?

What does this tell you about $\cos \theta^\circ$ when $90^\circ < \theta^\circ < 180^\circ$?

(b) What can you say about $\sin \theta^\circ$ when $90^\circ < \theta^\circ < 180^\circ$?

A5 Use the unit circle to show that $\sin 20^\circ = \sin 160^\circ$.

A6 Use the unit circle to show that $\sin(180^\circ - \theta^\circ) = \sin \theta^\circ$ for all values of θ° between 0° and 180° .

A7 Draw a sketch graph of $y = \sin \theta^\circ$ for $0^\circ < \theta^\circ < 180^\circ$.

A8 Use the unit circle to show that $\cos 20^\circ = -\cos 160^\circ$.

- A9** Use the unit circle to show that $\cos(180^\circ - \theta^\circ) = -\cos \theta^\circ$ for all values of θ° between 0° and 180° .
- A10** Draw a sketch graph of $y = \cos \theta^\circ$ for $0^\circ < \theta^\circ < 180^\circ$.
- A11** Use a calculator to find (to the nearest degree) the acute angle θ° such that $\sin \theta^\circ = 0.63$. Use your previous results to find another angle (less than 360°) whose sine is 0.63.
- A12** Use a calculator to find (to the nearest degree) the acute angle θ° such that $\cos \theta^\circ = 0.63$. Use your previous results to find an obtuse angle whose cosine is -0.63 .

Example 1

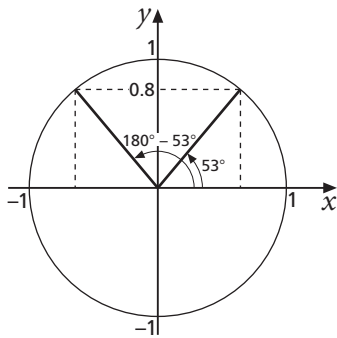
Find two angles (to the nearest degree) in the range $0^\circ < \theta^\circ < 180^\circ$ such that $\sin \theta^\circ = 0.8$.

Solution

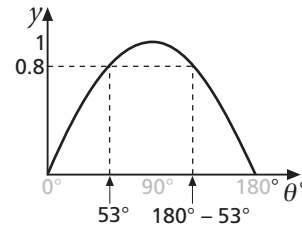
Key in $\sin^{-1} 0.8$

$\theta^\circ = 53^\circ$ is one angle.

To obtain the other angle think about the unit circle ...



... or the graph of $\sin \theta^\circ$.



The other angle is $180^\circ - 53^\circ = 127^\circ$.

Exercise A (answers p 138)

- 1 State whether each of these is positive or negative.

(a) $\sin 95^\circ$	(b) $\cos 95^\circ$	(c) $\sin 170^\circ$	(d) $\cos 170^\circ$
---------------------	---------------------	----------------------	----------------------
- 2 (a) Use your calculator to find (to the nearest degree) the angle between 0° and 90° whose sine is 0.34.
 (b) Hence find the two solutions to $\sin \theta^\circ = 0.34$ ($0^\circ < \theta^\circ < 180^\circ$).
- 3 Solve each of these equations, giving solutions between 0° and 180° to the nearest degree.

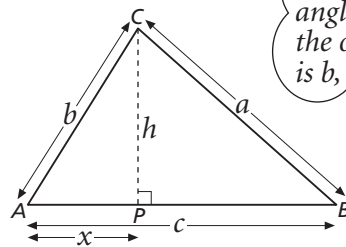
(a) $\sin \theta^\circ = 0.9$	(b) $\cos \theta^\circ = 0.9$	(c) $\sin \theta^\circ = 0.45$	(d) $\cos \theta^\circ = -0.45$
(e) $\sin \theta^\circ = 0.53$	(f) $\cos \theta^\circ = 0.53$	(g) $\sin \theta^\circ = 0.07$	(h) $\cos \theta^\circ = -0.07$
- *4 Explain why, if $0^\circ < \theta^\circ < 180^\circ$ and $-1 < k < 1$, $\sin \theta^\circ = k$ has either no solution or two solutions for θ° , but $\cos \theta^\circ = k$ always has one solution.

B Cosine rule (answers p 138)

It is easy to 'solve' a right-angled triangle (find its unknown angles and sides) by using Pythagoras's theorem and the sine, cosine and tangent ratios. But if a triangle is not right-angled, we need other methods.

B1 Consider triangle ABC .

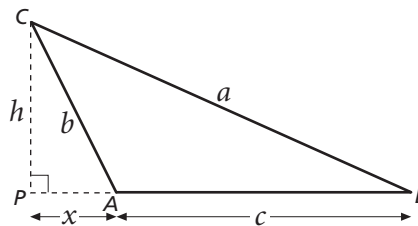
- Write PB in terms of c and x .
- Use Pythagoras in triangle CPB to write an expression for h^2 in terms of a , x and c .
- Use Pythagoras in triangle CPA to write an expression for h^2 in terms of b and x .
- Put your two expressions for h^2 equal to each other, and make a^2 the subject.
- Expand brackets and simplify this formula for a^2 .
- In triangle CPA , write an expression for x in terms of b and angle A .
- Substitute the expression for x in the formula you obtained in part (e). Check that you have obtained $a^2 = b^2 + c^2 - 2bc \cos A$.



The side opposite angle A is called a , the one opposite B is b , and so on.

B2 Prove that the rule you obtained in B1 is still true when angle A is obtuse.

(Hint: you will need to use the fact that $\cos(180^\circ - \theta) = -\cos \theta$.)



K The **cosine rule** states that $a^2 = b^2 + c^2 - 2bc \cos A$.

Similarly, $b^2 = a^2 + c^2 - 2ac \cos B$ and $c^2 = a^2 + b^2 - 2ab \cos C$.

Example 2

Find the length of side GF in triangle EFG .

Solution

The triangle is not right-angled. You know two sides and the enclosed angle, so use the cosine rule.

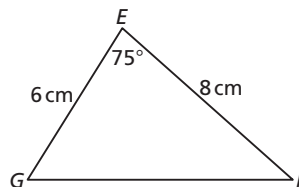
In triangle EFG , $e^2 = f^2 + g^2 - 2fg \cos E$

$$e^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 75^\circ$$

$$e^2 = 75.153\dots$$

$$e = 8.67 \text{ (to 2 d.p.)}$$

So the length of GF is 8.67 cm (to 2 d.p.).

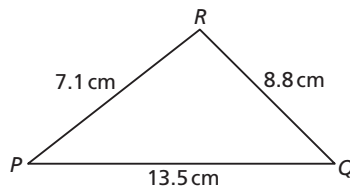


Example 3

Find angle R in triangle PQR .

Solution

You know all three sides of the triangle, so use the cosine rule.



In triangle PQR , $r^2 = p^2 + q^2 - 2pq \cos R$

$$13.5^2 = 8.8^2 + 7.1^2 - 2 \times 8.8 \times 7.1 \times \cos R$$

$$\Rightarrow 2 \times 8.8 \times 7.1 \times \cos R = 8.8^2 + 7.1^2 - 13.5^2$$

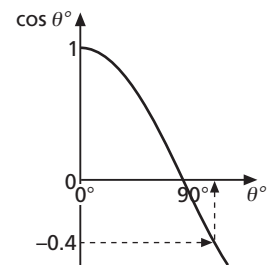
$$\Rightarrow \cos R = \frac{8.8^2 + 7.1^2 - 13.5^2}{2 \times 8.8 \times 7.1}$$

$$\text{so } \cos R = -0.4353\dots$$

$$R = 115.8\dots^\circ$$

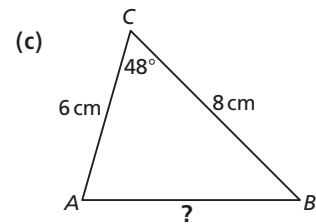
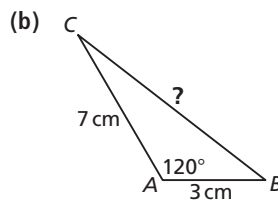
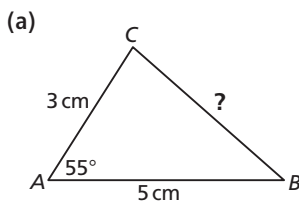
$$= 116^\circ \text{ to the nearest degree}$$

The cosine is negative, so the angle is obtuse.

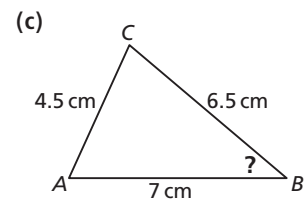
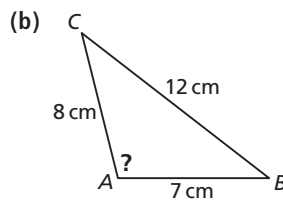
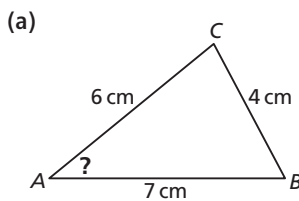


Exercise B (answers p 138)

1 Work out the length marked ? in each of these triangles.



2 Work out the angle marked ? in each of these triangles.



3 The hands of a clock are 10 cm and 7 cm long.

Calculate the distance between their tips when it is exactly 2 o'clock.

4 A triangle has sides 4 cm, 5 cm and 7 cm.

Calculate all its angles.

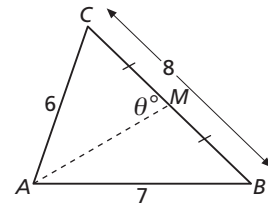
5 A is 2.1 km due north of B.

C is 3.7 km from B on a bearing of 136° .

Find the distance from C to A.

*6 In the triangle shown, $BC = 8$, $CA = 6$, $AB = 7$ and M is the mid-point of BC . Angle $AMC = \theta^\circ$.

- Use the cosine rule to write down an expression for AC^2 in terms of $\cos \theta^\circ$, CM and AM .
- Write an expression for AB^2 in terms of $\cos \theta^\circ$, BM and AM .
- Add the expressions you obtained in (a) and (b), and hence calculate the length of AM .

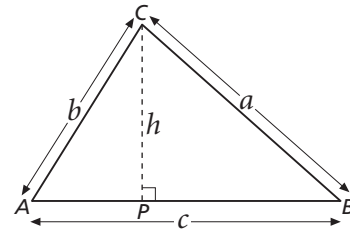


C Sine rule (answers p 138)

The cosine rule enables you to find a side when you know two sides and the angle they enclose, or an angle when you know all three sides.

You can solve other triangles using the **sine rule**.

- In triangle APC , write h in terms of b and angle A .
 - In triangle BPC , write h in terms of a and angle B .
 - Put these two expressions for h equal to each other.
 - Divide both sides of your equation in (c), first by $\sin A$, and then by $\sin B$.
 - Check that your result is $\frac{a}{\sin A} = \frac{b}{\sin B}$



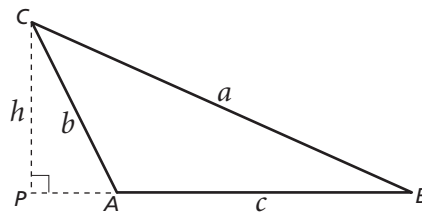
If we start with sides b and c and angles B and C , we can prove that $\frac{b}{\sin B} = \frac{c}{\sin C}$.

K Combining these two results we have the **sine rule**:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

C2 In triangle ABC , angle A is obtuse.

- In triangle PAC , express h in terms of b and angle PAC .
- Express angle PAC in terms of angle A (i.e. angle CAB).
Rewrite the expression for h you obtained in (a) in terms of angle A .
- Use the fact that $\sin(180^\circ - \theta^\circ) = \sin \theta^\circ$ to simplify your expression.
- Now write an expression for h in terms of side a and angle B .
- Hence show that in this triangle $\frac{a}{\sin A} = \frac{b}{\sin B}$.



Since angles B and C are acute, $\frac{b}{\sin B} = \frac{c}{\sin C}$, as before. So $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

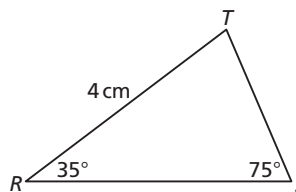
Example 4

Find the length of side ST in triangle RST .

Solution

You know side RT and the angle opposite it.

You know the angle opposite side ST , so use the sine rule.



In triangle RST , $\frac{r}{\sin R} = \frac{s}{\sin S} = \frac{t}{\sin T}$

You don't need the $\frac{t}{\sin T}$ part and could omit it.

$$\frac{r}{\sin 35^\circ} = \frac{4}{\sin 75^\circ}$$

$$\Rightarrow r = \frac{4 \times \sin 35^\circ}{\sin 75^\circ} = 2.375\dots$$

so $ST = 2.4$ cm (to 1 d.p.)

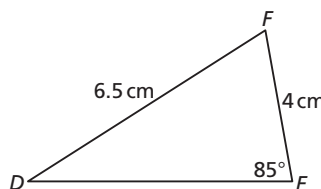
Example 5

Find angle D in triangle DEF .

Solution

You know side e and angle E .

You know side d , so use the sine rule.



In triangle DEF , $\frac{d}{\sin D} = \frac{e}{\sin E}$

$$\frac{4}{\sin D} = \frac{6.5}{\sin 85^\circ}$$

$$\Rightarrow 4 \times \sin 85^\circ = 6.5 \times \sin D$$

$$\Rightarrow \sin D = \frac{4 \times \sin 85^\circ}{6.5} = 0.613\dots$$

Use your calculator.

$D = 38^\circ$ (to the nearest degree)

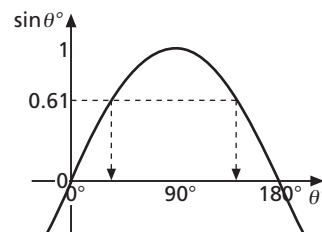
Another answer is therefore $D = 180^\circ - 38^\circ = 142^\circ$

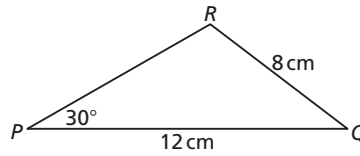
But if $D = 142^\circ$, then the angle sum of triangle DEF is clearly greater than 180° .

Hence the only possible solution here is

$D = 38^\circ$ (to the nearest degree).

$\sin \theta^\circ$ and $\sin (180^\circ - \theta^\circ)$ both have the same value.



Example 6Find angle R in triangle PQR .**Solution**In triangle PQR , $\frac{p}{\sin P} = \frac{r}{\sin R}$

$$\frac{8}{\sin 30^\circ} = \frac{12}{\sin R}$$

$$\Rightarrow 8 \sin R = 12 \sin 30^\circ$$

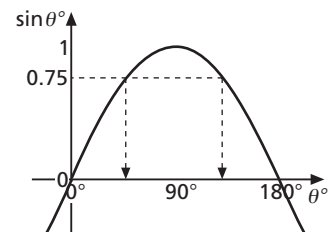
$$\Rightarrow \sin R = \frac{12 \sin 30^\circ}{8} = 0.75$$

Use your calculator. $R = 49^\circ$ (to the nearest degree)Another answer is therefore $R = 180^\circ - 49^\circ = 131^\circ$

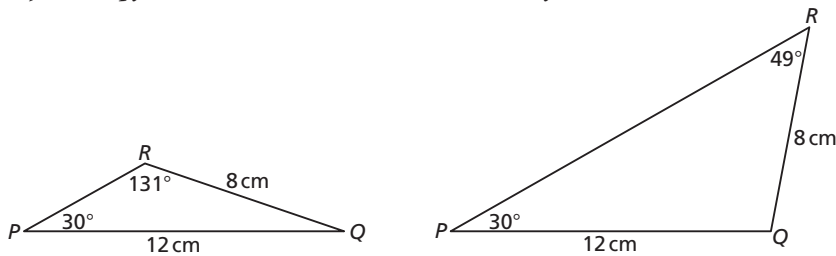
Check whether both answers are possible.

If $R = 49^\circ$, then $Q = 180^\circ - 30^\circ - 49^\circ = 101^\circ$ If $R = 131^\circ$, then $Q = 180^\circ - 30^\circ - 131^\circ = 19^\circ$

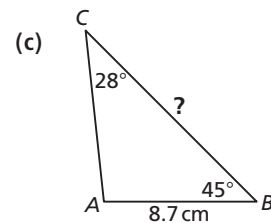
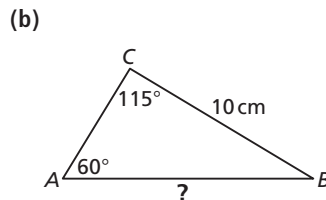
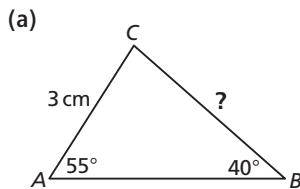
Both answers are possible.

So $R = 49^\circ$ or 131° (to the nearest degree). $\sin \theta^\circ$ and $\sin (180^\circ - \theta^\circ)$ both have the same value.

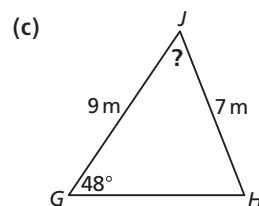
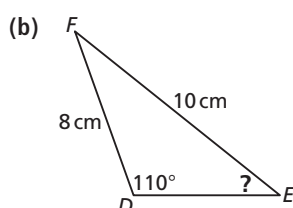
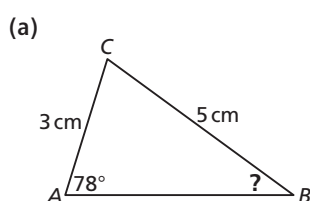
It may be helpful to sketch the two solutions as a further check.

**Exercise C** (answers p 138)

1 Find the length marked ? in each of these triangles.



- 2 In triangle ABC , $c = 11$ cm, $a = 12$ cm and angle $A = 68^\circ$.
- Draw a sketch of triangle ABC .
 - Use the sine rule to calculate the value of $\sin C$.
 - Your result in (b) can lead to two different values for angle C . What are these (to the nearest degree)?
 - For each of your possible values of angle C , find the value of angle B .
 - Write down the value or values of angle C that are possible. For each possible value of C draw a rough sketch of the triangle.
- 3 Find the angle marked ? in each of these triangles. Where there is more than one solution, give both.



D Both rules

To solve many triangles you will need to use a rule twice, or use the sine rule and the cosine rule.

Example 7

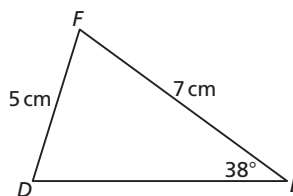
Solve triangle DEF .

Solution

'Solve' means find all the unknown angles and sides.

Decide which rule to use first.

In this case, the sine rule is possible.



In triangle DEF , $\frac{d}{\sin D} = \frac{e}{\sin E} = \frac{f}{\sin F}$

$$\frac{7}{\sin D} = \frac{5}{\sin 38^\circ}$$

$$\Rightarrow 7 \sin 38^\circ = 5 \sin D$$

$$\Rightarrow \sin D = \frac{7 \sin 38^\circ}{5} = 0.8619\dots$$

$\frac{f}{\sin F}$ is not needed.

Use the calculator. $D = 59.53\dots^\circ = 59.5^\circ$ (to 1 d.p.)

Another answer is therefore $D = 180^\circ - 59.5^\circ = 120.5^\circ$

Check whether both answers are possible.

$$\text{If } D = 59.5^\circ \text{ then } F = 180^\circ - 38^\circ - 59.5^\circ = 82.5^\circ$$

$$\text{If } D = 120.5^\circ \text{ then } F = 180^\circ - 38^\circ - 120.5^\circ = 21.5^\circ$$

Both answers are possible. So $D = 59.5^\circ$ or 120.5° (to 1 d.p.)

(Solution continues over.)

Deal with each possible value of D separately.
For each value, draw a sketch showing what you know so far.

Use the result you obtained before rounding.

If $D = 59.53\dots^\circ$

then $F = 180^\circ - 38^\circ - 59.53\dots^\circ = 82.46\dots^\circ$

Now use the sine rule again.

$$\frac{f}{\sin 82.46\dots^\circ} = \frac{5}{\sin 38^\circ}$$

$$\Rightarrow f = \frac{5 \sin 82.46\dots^\circ}{\sin 38^\circ} = 8.05\dots = 8.1 \text{ (1 d.p.)}$$

So the first solution is $D = 59.5^\circ$, $F = 82.5^\circ$ and $DE = 8.1$ cm (each answer to 1 d.p.).

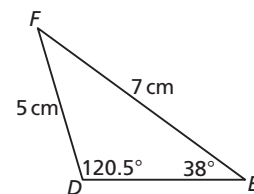
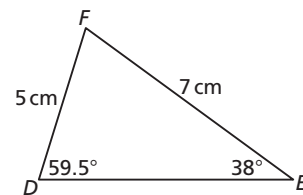
Alternatively, if $D = 180^\circ - 59.53\dots^\circ = 120.46\dots^\circ$
then $F = 180^\circ - 38^\circ - 120.46\dots^\circ = 21.53\dots^\circ$

and

$$\frac{f}{\sin 21.53\dots^\circ} = \frac{5}{\sin 38^\circ}$$

$$\Rightarrow f = \frac{5 \sin 21.53\dots^\circ}{\sin 38^\circ} = 2.98\dots = 3.0 \text{ (1 d.p.)}$$

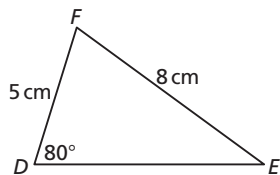
So the second solution is $D = 120.5^\circ$, $F = 21.5^\circ$ and $DE = 3.0$ cm (answers to 1 d.p.).



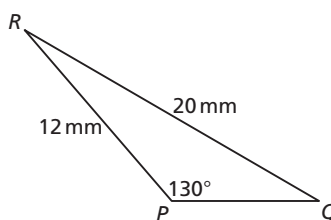
Exercise D (answers p 139)

- 1 Use the sine rule to solve (find all the unknown angles and sides in) each of these triangles.
If there is more than one solution, give both solutions.

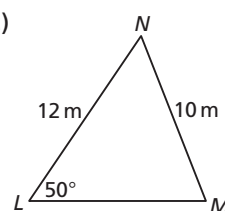
(a)



(b)

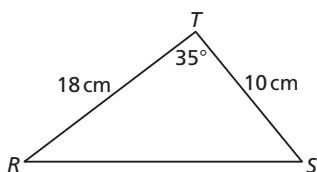


(c)

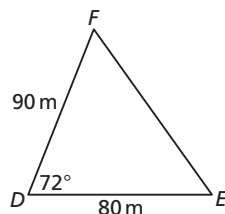


- 2 Use the cosine rule and then the sine rule to solve these triangles.

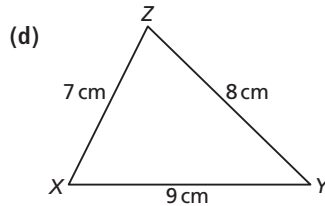
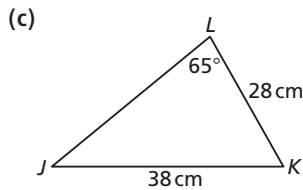
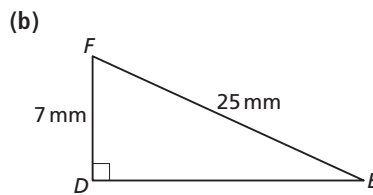
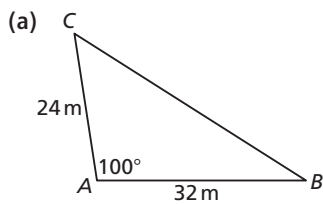
(a)



(b)



3 Solve these triangles.



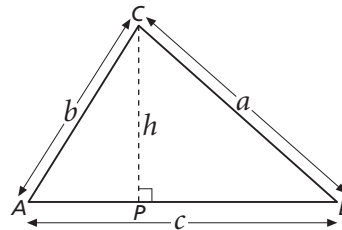
E Area of a triangle

There is a simple formula for the area of a triangle, when you know two sides and the angle between them.

In triangle PBC , $h = a \sin B$.

$$\begin{aligned} \text{The area of triangle } ABC &= \frac{1}{2} \text{ base} \times \text{height} \\ &= \frac{1}{2} c \times h = \frac{1}{2} c \times a \sin B \\ &= \frac{1}{2} ac \sin B \end{aligned}$$

By drawing perpendiculars to the other two sides we can show that:



K Area of triangle ABC
 $= \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B$

This result also holds true for an obtuse-angled triangle.

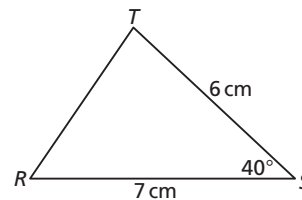
Example 8

Find the area of triangle RST .

Solution

$$\begin{aligned} \text{Area of triangle } RST &= \frac{1}{2} rt \sin S \\ &= \frac{1}{2} \times 6 \times 7 \times \sin 40^\circ \\ &= 13.498\dots \end{aligned}$$

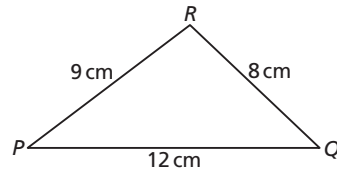
You know S , so the lengths in the formula are r and t .



The area of triangle RST is 13.5 cm^2 (to 1 d.p.).

Example 9Find the area of triangle PQR .**Solution**

To find the area you need two sides and the included angle.
So first find an angle, using the cosine rule.
Any angle will do; here angle P is found.



$$p^2 = q^2 + r^2 - 2qr \cos P$$

$$8^2 = 9^2 + 12^2 - 2 \times 9 \times 12 \times \cos P$$

$$\cos P = \frac{9^2 + 12^2 - 8^2}{2 \times 9 \times 12} = 0.745\dots$$

$$P = 41.8\dots^\circ$$

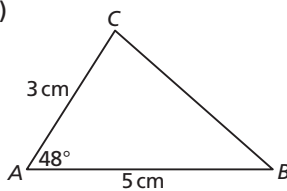
Don't lose any accuracy at this stage.

$$\begin{aligned} \text{So area of triangle } PQR &= \frac{1}{2}qr \sin P \\ &= \frac{1}{2} \times 9 \times 12 \times \sin 41.8\dots^\circ \\ &= 35.9991\dots \end{aligned}$$

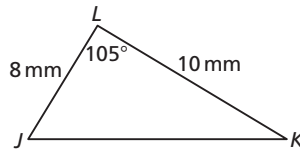
So the area is 36 cm^2 (to the nearest cm^2).

Exercise E (answers p 139)**1** Find the area of each triangle.

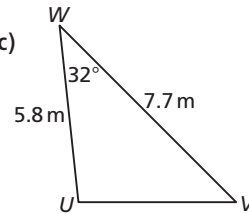
(a)



(b)



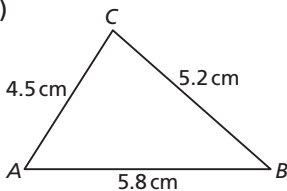
(c)

**2** Triangle RST has $RS = 8 \text{ cm}$, $ST = 10 \text{ cm}$ and $TR = 11 \text{ cm}$.

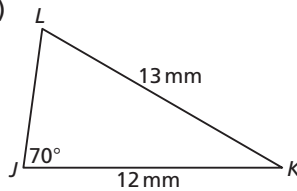
- (a) Use the cosine rule to find one angle of the triangle.
(b) Use your result to find the area of the triangle.

3 Find the area of each of these triangles.

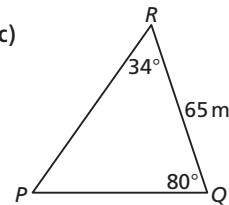
(a)



(b)



(c)



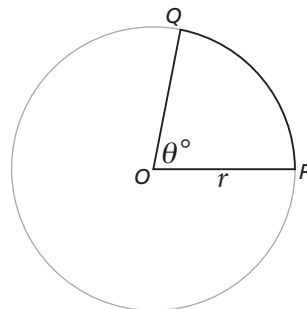
F Radians and arcs (answers p 139)

We measure angles in degrees for historical reasons: the ancient Babylonians divided their day into 360 units of time, and their circle into 360 units too.

But using the degree as the unit of angle measurement leads to complicated formulas.

For example, the distance all round the circle shown is $2\pi r$.

The arc PQ , which subtends an angle of θ degrees at the centre, is therefore $\frac{\theta}{360}$ of the whole circle, or $\frac{\theta}{360} \times 2\pi r = \frac{\pi r \theta}{180}$.



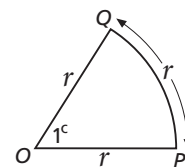
To simplify this formula we use a different measure of angle in more advanced mathematics.

This measure is called the **radian**.

One radian is defined as the angle subtended at the centre of a circle radius r by an arc of length r (i.e. equal to the radius).

As an arc of length r subtends an angle of one radian, so the whole circumference of the circle ($2\pi r$) subtends 2π radians.

So $360^\circ = 2\pi$ radians.



F1 Write the following angles in radians, giving answers in terms of π .

- (a) 180° (b) 90° (c) 60° (d) 30° (e) 270° (f) 1°

F2 Work out the size of one radian in degrees.

F3 In a circle of radius r , an angle of one radian is subtended by an arc of length r .

- (a) What length arc subtends an angle of 2 radians?
 (b) What length arc subtends an angle of $\frac{1}{2}$ radian?
 (c) What length arc subtends an angle of θ radians?

F4 A circle has a radius of 8 cm.

An arc subtends an angle of 0.25 radians at the centre of the circle.
 How long is the arc, in centimetres?

Angles can be measured in radians, where 2π radians = 360° .

We can write 2π radians as $2\pi^c$ or 2π rad, but the c or rad is normally omitted.

K

Other important equivalents that you should remember are

$$180^\circ = \pi \text{ rad} \quad 90^\circ = \frac{\pi}{2} \text{ rad} \quad 60^\circ = \frac{\pi}{3} \text{ rad} \quad 45^\circ = \frac{\pi}{4} \text{ rad} \quad 30^\circ = \frac{\pi}{6} \text{ rad}$$

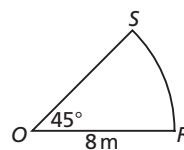
If an arc subtends an angle of θ radians at the centre of a circle radius r , the length of the arc is $r\theta$.

Example 10

A sector with angle 45° is cut from a circle of radius 8 metres.

Find

- (a) the arc length RS (b) the perimeter of the sector



Solution

To use the formula for arc length, you must work in radians.

(a) The arc length $RS = r\theta = 8 \times \frac{\pi}{4} = 6.283\dots$

Arc $RS = 6.3$ m (to 1 d.p.)

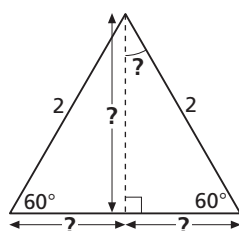
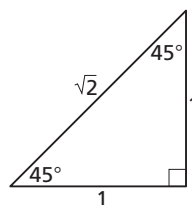
(b) Perimeter of sector = $OR + OS + \text{arc } RS = 8 + 8 + 6.283\dots = 22.283\dots$

Perimeter = 22.3 m (to 1 d.p.)

Some values of sine, cos and tan can be found easily using Pythagoras and symmetry.

A 45° right-angled triangle whose shorter sides are of length 1 unit has (by Pythagoras) a hypotenuse of length $\sqrt{2}$.

Hence $\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$ and $\tan 45^\circ = 1$.



F5 The diagram shows an equilateral triangle with a perpendicular from the apex to the base.

- (a) Copy the diagram and fill in the lengths and angle marked with ?.
- (b) Hence write down the sine, cos and tan of both 30° and 60° , giving them in surd form.

K

You should know or be able to quickly work out that

$$\sin \frac{\pi}{4} = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos \frac{\pi}{4} = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\tan \frac{\pi}{4} = \tan 45^\circ = 1$$

$$\sin \frac{\pi}{6} = \sin 30^\circ = \frac{1}{2}$$

$$\cos \frac{\pi}{6} = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan \frac{\pi}{6} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin \frac{\pi}{3} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{3} = \cos 60^\circ = \frac{1}{2}$$

$$\tan \frac{\pi}{3} = \tan 60^\circ = \sqrt{3}$$

Exercise F (answers p 139)

1 Change the following angles into radians, giving answers in terms of π .

- (a) 210° (b) 135° (c) 120° (d) 330° (e) 300°

2 Change the following angles into degrees.

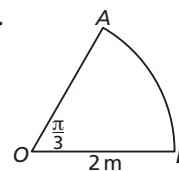
- (a) $\frac{\pi}{8}$ (b) $\frac{\pi}{10}$ (c) $\frac{\pi}{180}$ (d) $\frac{\pi}{4}$ (e) $\frac{5\pi}{6}$
 (f) $\frac{5\pi}{4}$ (g) $\frac{5\pi}{12}$ (h) $\frac{7\pi}{4}$ (i) $\frac{2\pi}{9}$ (j) $\frac{4\pi}{3}$

3 Copy this table and complete it, giving exact values for sine, cos and tan. All angles are between 0 and π radians.

Radians	Degrees	$\sin \theta$	$\cos \theta$	$\tan \theta$
$\frac{\pi}{6}$		$\frac{1}{2}$		$\frac{1}{\sqrt{3}}$
$\frac{2\pi}{3}$				
	135°			
$\frac{5\pi}{6}$				

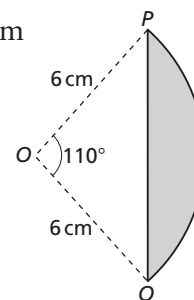
4 The sector OAB is cut from a circle of radius 2 m.

- (a) What is the length of arc AB ?
 (b) What is the perimeter of the sector?



5 The curved edge of the pendant shown shaded in the diagram is an arc of a circle, radius 6 cm, which subtends an angle of 110° at the centre of the circle.

- (a) Calculate the length of the arc.
 (b) Calculate the length of the straight line PQ .
 (c) Hence work out, to 3 s.f., the perimeter of the pendant.



6 A circle has radius 7 cm.

An arc of the circle has length 10 cm.

What angle, in degrees, does the arc subtend at the centre of the circle?

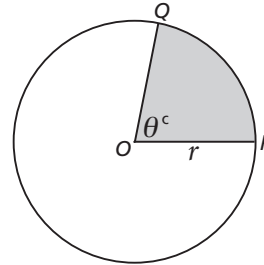
7 A sector of a circle has an angle at the centre of $\frac{\pi}{4}$ and a perimeter of 12 cm. Work out the radius of the circle.

G Area of a sector

The area of a complete circle of radius r is πr^2 .

The sector POQ occupies $\frac{\theta}{2\pi}$ of the whole circle.

Hence the area of sector POQ is $\frac{\theta}{2\pi} \times \pi r^2$ or $\frac{1}{2}r^2\theta$.



K The area of a sector of a circle, radius r , that subtends an angle θ at the centre is $\frac{1}{2}r^2\theta$.

Example 11

A sector with angle 55° is cut from a circle of radius 8 metres. Find the area of the sector.

Solution

To use the formula above, work in radians.

$$\begin{aligned} 55^\circ &= \frac{\pi}{180} \times 55 \text{ radians, so the area of the sector is } \frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times \frac{\pi}{180} \times 55 \\ &= 30.71\dots \end{aligned}$$

The area is 30.7 m^2 (to 1 d.p.).

Example 12

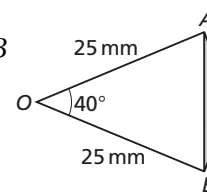
A circle, centre O , has radius 25 mm. A sector OAB of the circle subtends an angle at O of 40° .

Find the area of the segment bounded by the arc AB and the chord AB .

Solution

Draw a sketch.

Area of segment (shaded) = area of sector OAB – area of triangle OAB



To find the area of the sector OAB , work in radians.

$$40^\circ = \frac{\pi}{180} \times 40 \text{ radians}$$

$$\text{So area of sector } OAB = \frac{1}{2} \times 25^2 \times \frac{\pi}{180} \times 40 \text{ mm}^2 = 218.16\dots \text{ mm}^2 \quad \text{Do not approximate yet.}$$

$$\text{Area of triangle } OAB = \frac{1}{2} \times 25 \times 25 \times \sin 40^\circ = 200.87\dots \text{ mm}^2$$

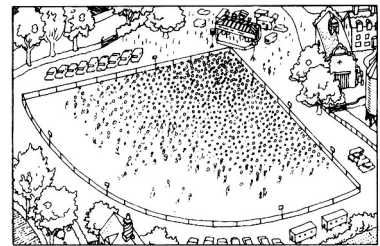
$$\begin{aligned} \text{Area of segment} &= 218.16\dots \text{ mm}^2 - 200.87\dots \text{ mm}^2 \\ &= 17.29\dots \text{ mm}^2 = 17.3 \text{ mm}^2 \text{ (to 1 d.p.)} \end{aligned}$$

You can use radians or degrees here, as long as your calculator is in the corresponding mode.

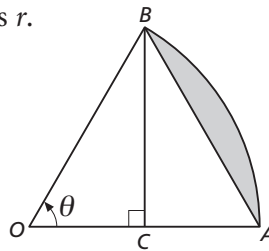
Exercise G (answers p 139)

- 1 Find the areas to 1 d.p. of sectors with
 - (a) angle 115° and radius 9 cm
 - (b) angle 20° and radius 20 m
 - (c) angle 200° and radius 5.5 cm
 - (d) angle 19° and radius 45 mm
- 2 Find the areas of these sectors in terms of π .
 - (a) angle $\frac{\pi}{4}$, radius 8
 - (b) angle 120° , radius 10
- 3 A sector has an area of 20 cm^2 and radius 8 cm.
What angle, in degrees, does it subtend at the centre?
- 4 A sector has radius r cm and angle at the centre of θ radians.
The perimeter of the sector is 18 cm.
 - (a) Find an expression for θ in terms of r .
 - (b) Find an expression for the area of the sector in terms of r .

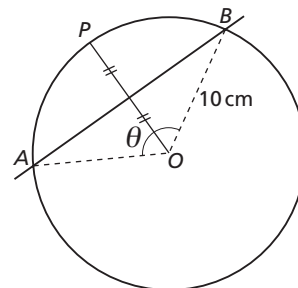
- 5 An area in the shape of a sector is to be fenced off for a crowd at a concert. The sector has radius 400 m, and angle at the centre of 2° .
 - (a) Calculate the length of fence needed for the perimeter.
 - (b) Health and Safety inspectors decide that the crowd density should not exceed 1 person per 2 m^2 . Calculate the maximum crowd.



- 6 OAB is a sector of a circle, centre O , radius r . Find these in terms of r and θ .
 - (a) The length BC
 - (b) The area of triangle OAB
 - (c) The area of the sector OAB
 - (d) The area of the shaded segment

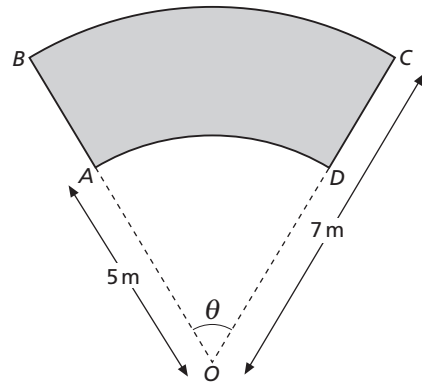


- 7 A circular cake of diameter 20 cm is cut along AB , halfway from the centre to the rim.
 - (a) Show that the angle θ is 120° .
 - (b) Calculate the area of the sector OAB , to 1 d.p.
 - (c) Work out the area of triangle OAB , to 1 d.p.
 - (d) Hence find the area of cake cut off, to 1 d.p.



- 8 The diagram shows a gardener's design for the shape of a flower bed with perimeter $ABCD$.

AD is an arc of a circle with centre O and radius 5 m.
 BC is an arc of a circle with centre O and radius 7 m.
 OAB and ODC are straight lines and the size of $\angle AOD$ is θ radians.



- (a) Find, in terms of θ , an expression for the area of the flower bed.

Given that the area of the flower bed is 15 m^2 ,

- (b) show that $\theta = 1.25$,
 (c) calculate, in m, the perimeter of the flower bed.

The gardener now decides to replace arc AD with the straight line AD .

- (d) Find, to the nearest cm, the reduction in the perimeter of the flower bed.

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Key points

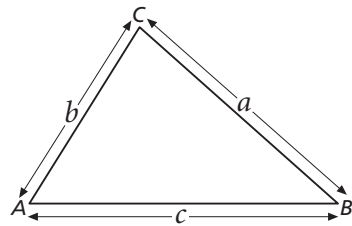
- When solving a triangle ABC you can use the cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

the sine rule:
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



(pp 54, 56)

- The area of any triangle ABC is given by $\frac{1}{2}ab \sin C = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B$ (p 61)

- Angles can be measured in radians, where 2π radians = 360° .

Other important equivalents that you should remember are

$$180^\circ = \pi \text{ rad} \quad 90^\circ = \frac{\pi}{2} \text{ rad} \quad 60^\circ = \frac{\pi}{3} \text{ rad} \quad 45^\circ = \frac{\pi}{4} \text{ rad} \quad 30^\circ = \frac{\pi}{6} \text{ rad} \quad (\text{p } 63)$$

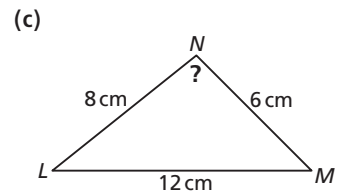
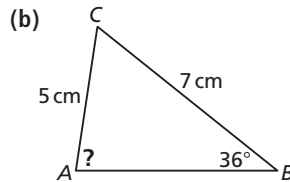
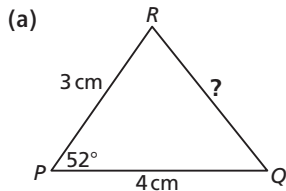
- If an arc subtends an angle of θ radians at the centre of a circle radius r , the length of the arc is $r\theta$. (p 63)

- The area of a sector of a circle, radius r , with angle θ radians at the centre is $\frac{1}{2}r^2\theta$. (p 66)

Test yourself (answers p 140)

- 1 Work out the value of ? in each triangle below.

Where there is more than one value, give both.



- 2 Find the area of each triangle in question 1.

Give both answers if there are two.

- 3 A sector of a circle, radius 5 cm, subtends 78° at the centre.

Calculate the sector's

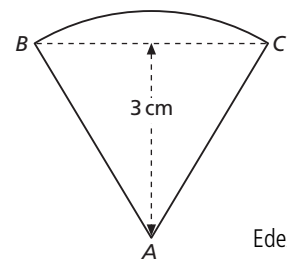
- (a) arc length (b) perimeter (c) area

- 4 A sector of a circle has radius 12 cm and area 90 cm^2 .

What is the perimeter of the sector?

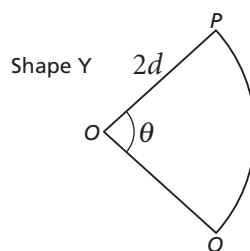
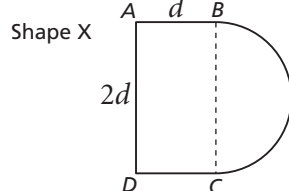
- 5 The shape of a badge is a sector ABC of a circle with centre A and radius AB , as shown in the diagram. The triangle ABC is equilateral and has perpendicular height 3 cm.

- (a) Find, in surd form, the length of AB .
 (b) Find, in terms of π , the area of the badge.
 (c) Prove that the perimeter of the badge is $\frac{2\sqrt{3}}{3}(\pi + 6) \text{ cm}$.



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- 6



The diagram above shows the cross-sections of two drawer handles.

Shape X is a rectangle $ABCD$ joined to a semicircle with BC as diameter.

The length $AB = d \text{ cm}$ and $BC = 2d \text{ cm}$.

Shape Y is a sector OPQ of a circle with centre O and radius $2d \text{ cm}$.

Angle POQ is θ radians.

Given that the areas of shapes X and Y are equal,

- (a) prove that $\theta = 1 + \frac{1}{4}\pi$.

Using this value of θ , and given that $d = 3$, find in terms of π ,

- (b) the perimeter of shape X (c) the perimeter of shape Y

- (d) Hence find the difference, in mm, between the perimeters of shapes X and Y. Edexcel