Core 1 for Edexcel contents

7 Polynomials

In this chapter you will learn how to
- evaluate and manipulate powers of the form kx^n, where n is a positive integer
- sketch cubic graphs and consider the effect of translations on their equations
- multiply out brackets to give a polynomial expression
- relate the roots of a polynomial to where its graph crosses the x-axis
- use function notation

A Indices: revision

Example 1

Evaluate $5x^3$ when $x = 2$.

Solution

When $x = 2$, $5x^3 = 5 \times 2^3$ *This is not the same as* $(5 \times 2)^3$.

$= 5 \times 8$

$= 40$

Example 2

Simplify $2x \times 3x^2$.

Solution

$$\begin{aligned} 2x \times 3x^2 &= 2 \times x \times 3 \times x \times x \\ &= 2 \times 3 \times x \times x \times x \\ &= 6x^3 \end{aligned}$$

Exercise A (answers p 181)

1 Evaluate these when $n = 3$.

(a) $n^2 + n^3$ **(b)** $2n^2$ **(c)** $(2n)^2$ **(d)** $(n + 2)^3$ **(e)** $3(n + 1)^2$
(f) $3n^2 - 5n$ **(g)** $2n^5 - 3n$ **(h)** $\frac{1}{3}n^4 + 1$ **(i)** $2(n^3 - 7)$ **(j)** $(n - 1)^3 + n^3$

2 Evaluate these when $x = \frac{1}{2}$.

(a) $6x^2$ **(b)** $(6x)^2$ **(c)** $5x(2x - 1)$ **(d)** $(x - 1)^3$

3 Evaluate these when $a = -1$.

(a) $10a^4$ **(b)** a^9 **(c)** $5a^5$ **(d)** $4(a + 3)^3$

4 Simplify each of these.

(a) $5x \times x^2$ **(b)** $4x^2 \times 3x$ **(c)** $2x^2 \times 5x^3$ **(d)** $\frac{1}{4}x^3 \times 8x$
(e) $5x^2 \times \frac{1}{3}x$ **(f)** $(2x)^3$ **(g)** $(3x^2)^2$ **(h)** $(\frac{1}{2}x^3)^2$

B Cubic graphs (answers p 181)

B1 Use a graph plotter on a computer or graphic calculator to draw some graphs of the form $y = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants.
Include some where the value of a is negative.
What do you notice about the shapes of your graphs?

B2 Use a graph plotter to draw each graph below.
How many times does each one cross or touch the x-axis?

$y = x^3 + 3x^2 + 4x + 5$ $\qquad y = x^3 - 2x^2 - x + 2$ $\qquad y = x^3 - x^2 - x + 1$

B3 Use a graph plotter to draw graphs of the three equations below.

$y = x^3$ $\qquad y = x^3 - x$ $\qquad y = x^3 + x$

Describe the shape of each graph as fully as you can.
Make sketches that show clearly the shape of each graph for $-1 \le x \le 1$.

B4 For each equation below

(i) make a sketch to show what you think the graph will look like

(ii) check by graphing on a graph plotter

(a) $y = -x^3$ $\qquad$ **(b)** $y = x^3 + 2$ $\qquad$ **(c)** $y = -x^3 - 5$

B5 **(a)** Make a sketch of $y = x^3$.

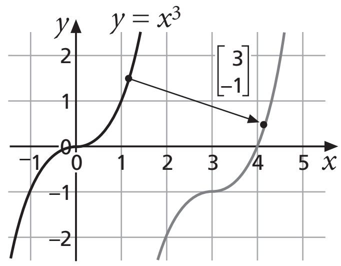
(b) Sketch what you think the graph of $y = (x - 2)^3$ will look like.

(c) Check on a graph plotter.

B6 For each equation below, sketch what you think the graph will look like and then check on a graph plotter.

(a) $y = (x + 4)^3$ $\qquad$ **(b)** $y = (x - 2)^3 + 5$ $\qquad$ **(c)** $y = (x + 6)^3 - 7$

B7 The graph of $y = x^3$ is translated by $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ as shown in the diagram.



(a) Which do you think is the correct equation for the new graph?

A $y = (x - 3)^3 + 1$ $\qquad$ **B** $y = (x - 3)^3 - 1$

C $y = (x + 3)^3 + 1$ $\qquad$ **D** $y = (x + 3)^3 - 1$

(b) Check on a graph plotter.

(c) Work out where the new graph crosses the y-axis.

B8 The graph of $y = x^3$ is translated by $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$.

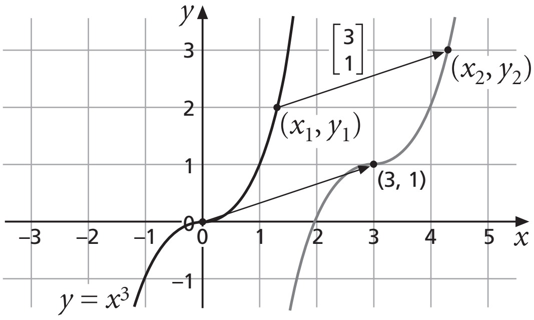
(a) Write down what you think is the correct equation for the translated graph.

(b) Work out where the translated graph crosses the y-axis.

We can work algebraically when translating curves.

In the diagram below, the graph of $y = x^3$ has been translated 3 units to the right and 1 unit up.

Using vector notation this translation is represented by $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$.



Let (x_1, y_1) be a point on $y = x^3$ and let (x_2, y_2) be its image on the translated curve.

Then $(x_2, y_2) = (x_1 + 3, y_1 + 1)$ giving

$x_1 = x_2 - 3$ *from rearranging* $x_2 = x_1 + 3$

and $y_1 = y_2 - 1$ *from rearranging* $y_2 = y_1 + 1$

We know that $y_1 = x_1^3$, so it must be true that

$y_2 - 1 = (x_2 - 3)^3$

and so $y - 1 = (x - 3)^3$ is the equation of the transformed curve.

This can be written as $y = (x - 3)^3 + 1$.

> So to find the equation of the curve that is the result of translating $y = x^3$ by $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ replace x by $(x - 3)$ and y by $(y - 1)$.

B9 Which translation will transform the curve $y = x^3$ to $y = (x - 5)^3 + 3$?

B10 The graph of $y = x^3 + x$ is translated by $\begin{bmatrix} 2 \\ -9 \end{bmatrix}$.

(a) Which do you think is the correct equation for the new graph?

A $y = (x + 2)^3 + (x + 2) + 9$ **B** $y = (x + 2)^3 + (x + 2) - 9$

C $y = (x - 2)^3 + (x - 2) + 9$ **D** $y = (x - 2)^3 + (x - 2) - 9$

(b) Check by graphing on a graph plotter.

(c) Work out where the new graph crosses the y-axis.

K To find the equation of any curve after it has been translated by $\begin{bmatrix} p \\ q \end{bmatrix}$, replace x by $(x - p)$ and replace y by $(y - q)$.

Example 3

Sketch the graph of $y = (x + 2)^3 + 5$.

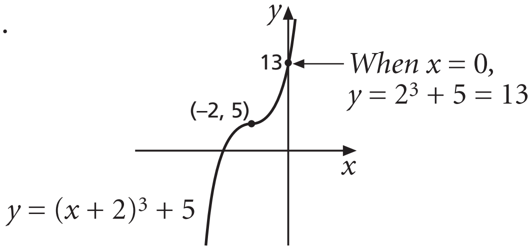
Solution

The equation can be written as $y - 5 = (x + 2)^3$.

This is $y = x^3$ with x replaced by $(x + 2)$ and y replaced by $(y - 5)$.

So the curve $y = x^3$ is translated by $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$.

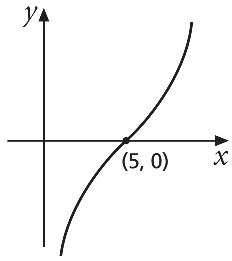
A sketch is shown on the right.



Example 4

The graph is a translation of $y = x^3 + x$ by $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$.

Find an equation for the graph.



Solution

The translation is $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ so replace x by $(x - 5)$, giving $y = (x - 5)^3 + (x - 5)$.

Exercise B (answers p 182)

1 Which translation will transform the curve $y = x^3$ to each of these?

(a) $y - 2 = (x - 1)^3$ **(b)** $y + 5 = (x - 3)^3$ **(c)** $y - 1 = (x + 7)^3$

2 Which translation will transform the curve $y = x^3$ to each of these?

(a) $y = (x - 4)^3 + 5$ **(b)** $y = (x + 2)^3 - 1$ **(c)** $y = (x + 6)^3 + 3$

3 The curve $y = x^3$ is translated by $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$.

(a) Write an equation for the translated curve.

(b) Sketch the curve, showing where it crosses each axis.

4 The curve $y = x^3$ is translated by $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$.

Write the equation of the translated curve in the form $y = (x + a)^3 + b$.

5 Find the equation of the curve $y = x^3 + x$ after a translation of $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

6 The curve $y = x^3 + x$ is translated by $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

(a) Write the equation for the translated curve in the form $y = (x + p)^3 + x + q$.

(b) Work out where the curve crosses the y-axis.

7 The curve $y = x^3 - x$ is translated by $\begin{bmatrix} -3 \\ -2 \end{bmatrix}$.

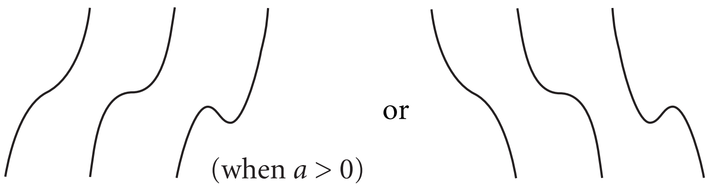
Write the equation for the translated curve in the form $y = (x + p)^3 - x + q$.

C Further graphs and manipulation (answers p 183)

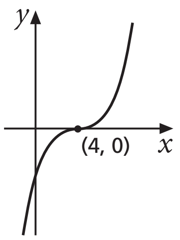
K

A **cubic** expression is one that can be written in the form $ax^3 + bx^2 + cx + d$ $(a \neq 0)$, where a, b, c and d are constants.

Graphs of the form $y = ax^3 + bx^2 + cx + d$ have one of these basic shapes:



Here is a sketch of $y = (x - 4)^3$.



You can multiply out the brackets to write $y = (x - 4)^3$ in the form $y = ax^3 + bx^2 + cx + d$.

$$\begin{aligned}(x-4)^3 &= (x-4)(x-4)(x-4)\\ &= (x-4)(x^2 - 4x - 4x + 16)\\ &= (x-4)(x^2 - 8x + 16)\end{aligned}$$

×	x	-4
x	x^2	$-4x$
-4	$-4x$	16

$$\begin{aligned} &= x^3 - 8x^2 + 16x - 4x^2 + 32x - 64\\ &= x^3 - 12x^2 + 48x - 64\end{aligned}$$

×	x^2	$-8x$	16
x	x^3	$-8x^2$	$16x$
-4	$-4x^2$	$32x$	-64

So the equation of the graph can also be written as $y = x^3 - 12x^2 + 48x - 64$.

The constant multipliers in an expression such as $x^3 - 12x^2 + 48x - 64$ are called **coefficients**. In this example, the coefficient of x^2 is -12.

C1 Write $y = (x + 3)^3$ in the form $y = ax^3 + bx^2 + cx + d$.

D **C2** Each expression below is a product of three different linear expressions. Multiply out the brackets in each one.

(a) $(x + 1)(x + 3)(x + 5)$ **(b)** $(2x + 1)(x + 2)(x - 3)$

(c) $2x(x - 4)(3x + 1)$ **(d)** $(5x - 1)(3x - 1)(x + 4)$

C3 **(a)** Solve the equation $(x + 3)(x + 2)(x - 1) = 0$.

(b) What does this tell you about the graph of $y = (x + 3)(x + 2)(x - 1)$?

(c) Where will the graph cross the y-axis?

(d) Sketch the graph of $y = (x + 3)(x + 2)(x - 1)$.

(e) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

C4 **(a)** Where will the graph of $y = x(x + 1)(x - 4)$ cross the x-axis?

(b) Sketch the graph of $y = x(x + 1)(x - 4)$.

(c) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

C5 **(a)** **(i)** Where will the graph of $y = (2x - 1)(x - 3)(x + 1)$ cross the x-axis?

(ii) Where will it cross the y-axis?

(b) Sketch the graph of $y = (2x - 1)(x - 3)(x + 1)$.

(c) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

D **C6** **(a)** Solve the equation $(x + 3)(x - 1)^2 = 0$.

(b) What does this tell you about the graph of $y = (x + 3)(x - 1)^2$?

(c) Where will the graph cross the y-axis?

(d) Sketch the graph of $y = (x + 3)(x - 1)^2$.

(e) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

C7 **(a)** Show that $(x + 3)^3 + 1$ is equivalent to $x^3 + 9x^2 + 27x + 28$.

(b) Sketch the graph of $y = x^3 + 9x^2 + 27x + 28$.

D **C8** **(a)** Copy and complete the statement $3x^3 - x^2 - 4x = x(\qquad\qquad)$.

(b) Factorise $3x^3 - x^2 - 4x$ completely.

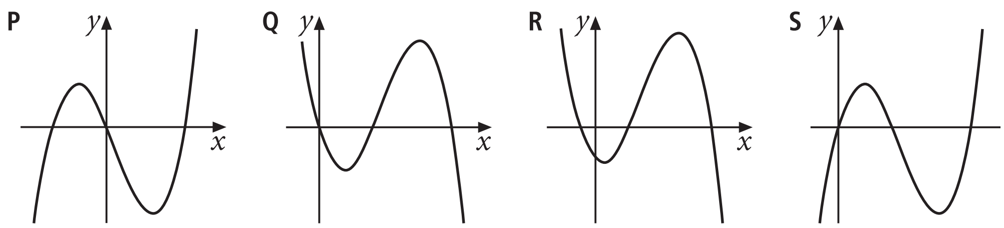
(c) Sketch the graph of $y = 3x^3 - x^2 - 4x$.

C9 **(a)** Solve the equation $x^2 - 2x - 4 = 0$.

(b) Hence solve $x^3 - 2x^2 - 4x = 0$ and sketch the graph of $y = x^3 - 2x^2 - 4x$.

C10 **(a)** Show that $x(x - 5)(2 - x)$ is equivalent to $-x^3 + 7x^2 - 10x$.

(b) Decide which of the graphs below is a sketch of $y = -x^3 + 7x^2 - 10x$.



K The product of three linear expressions can always be written in the form $ax^3 + bx^2 + cx + d$.

Example 5

Sketch the graph of $y = (2x - 1)(x - 3)(x + 2)$.

Solution

The expression is the product of three linear factors so the graph is a cubic shape.

The graph cuts the x-axis when $(2x - 1)(x - 3)(x + 2) = 0$.

The equation has three solutions:

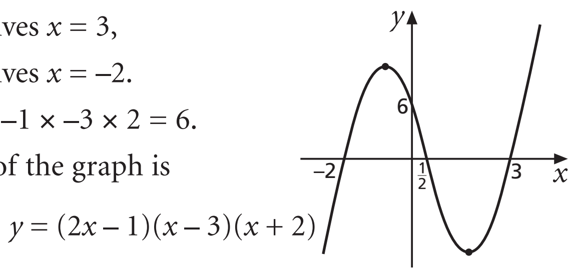
$2x - 1 = 0$ gives $x = \frac{1}{2}$,

$x - 3 = 0$ gives $x = 3$,

and $x + 2 = 0$ gives $x = -2$.

When $x = 0$, $y = -1 \times -3 \times 2 = 6$.

Hence, a sketch of the graph is



In Core 2, you will learn how to find the coordinates of the points marked with dots. Concentrate on the x- and y-intercepts for now.

Example 6

Write $(2x - 1)(x - 3)(x + 2)$ in the form $ax^3 + bx^2 + cx + d$.

Solution

$(2x - 1)(x - 3)(x + 2)$

$= (2x - 1)(x^2 + 2x - 3x - 6)$

$\times$	x	2
x	x^2	$2x$
-3	$-3x$	-6

$= (2x - 1)(x^2 - x - 6)$

$= 2x^3 - 2x^2 - 12x - x^2 + x + 6$

$= 2x^3 - 3x^2 - 11x + 6$

$\times$	x^2	$-x$	-6
$2x$	$2x^3$	$-2x^2$	$-12x$
-1	$-x^2$	x	6

Example 7

Solve the equation $2x^3 - 10x^2 + 12x = 0$.

Solution

$2x^3 - 10x^2 + 12x = 0$

2x is a factor of each term. $2x(x^2 - 5x + 6) = 0$

The quadratic factorises. $2x(x - 2)(x - 3) = 0$

So the solutions are $x = 0$, $x = 2$ and $x = 3$.

Exercise C (answers p 184)

1 For each equation below

  (i) sketch a graph, showing clearly where it crosses the x- and y-axes

  (ii) write the equation in the form $y = ax^3 + bx^2 + cx + d$

(a) $y = (x + 1)(x + 3)(x + 4)$ **(b)** $y = (x + 2)^3$

(c) $y = (x + 4)(x - 1)(x + 2)$ **(d)** $y = x(x + 3)(x - 2)$

(e) $y = (x - 5)^3$ **(f)** $y = (2x + 1)(x - 2)(x + 2)$

(g) $y = \frac{1}{2}x(2x + 3)(x - 2)$ **(h)** $y = (x + 4)^3 + 1$

2 **(a)** Solve the equation $(x + 1)(x - 3)^2 = 0$.

(b) Sketch the graph of $y = (x + 1)(x - 3)^2$, showing clearly where it meets both axes.

(c) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

3 **(a)** Copy and complete the statement $3x^3 - 3x^2 - 18x = 3x(\qquad\qquad)$.

(b) Factorise $3x^3 - 3x^2 - 18x$ completely.

(c) Sketch the graph of $y = 3x^3 - 3x^2 - 18x$, showing clearly where it crosses both axes.

4 **(a)** Factorise $x^3 + 5x^2 - 6x$ completely.

(b) Solve the equation $x^3 + 5x^2 - 6x = 0$.

5 Sketch the graph of $y = 2x^3 + 14x^2 + 24x$, showing clearly where it crosses both axes.

6 **(a)** Find the value of $y = (2x - 3)(x + 1)(3 - x)$ when $x = 1$.

(b) Sketch the graph of $y = (2x - 3)(x + 1)(3 - x)$.

(c) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

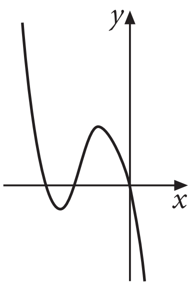
7 Solve the equation $5x^3 - 20x^2 + 20x = 0$.

8 **(a)** Solve the equation $x^3 - 10x^2 + 22x = 0$, giving each solution in surd form.

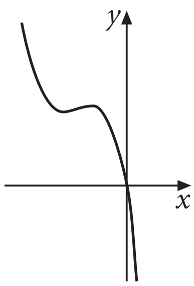
(b) Sketch the graph of $y = x^3 - 10x^2 + 22x$, showing where it crosses both axes.

9 Which of the graphs below is a sketch of $y = x^3 - 4x^2 + 5x$?

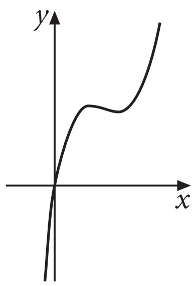
A



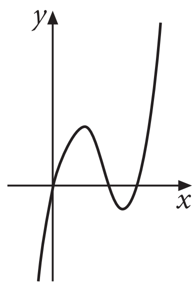
B



C



D



10 Sketch the graph of $y = (2x + 3)^2(x - 1)$, showing clearly where it meets both axes.

D Polynomial functions

K A **polynomial** is an expression that can be written in the form
$a + bx + cx^2 + dx^3 + ex^4 + fx^5 + \ldots$, where $a, b, c, d, e, f, \ldots$ are constants.

Some examples of polynomials are $7x^4 + x - 10$ and $6 + 5x^2 - x^3 - 5x^7$.

Expressions such as $\sqrt{x} + 3x^2 - 5$ and $\frac{1}{x} + 7x$ are not polynomials as they involve terms in x that cannot be written in the form x^n where n is a positive integer ($\sqrt{x}$ and $\frac{1}{x}$).

The **degree** of a polynomial is the value of its highest index.
For example, $6 + 5x^2 - x^3 - 5x^7$ is a polynomial of degree 7.

The sum, difference or product of two polynomials is also a polynomial.

For example,

$$\begin{aligned}&(x^4 - 5x)(3x^3 + x - 10)\\ &= 3x^7 + x^5 - 10x^4 - 15x^4 - 5x^2 + 50x\\ &= 3x^7 + x^5 - 25x^4 - 5x^2 + 50x\end{aligned}$$

×	$3x^3$	x	-10
x^4	$3x^7$	x^5	$-10x^4$
$-5x$	$-15x^4$	$-5x^2$	$50x$

Function notation is useful when dealing with polynomials.

For example, if a polynomial function is defined by $f(x) = x^3 + 4x^2 - 1$,
then f(3) is the value of the polynomial at $x = 3$.

$$\begin{aligned}\text{So } f(3) &= 3^3 + 4 \times 3^2 - 1\\ &= 27 + 36 - 1\\ &= 62\end{aligned}$$

Example 8

Polynomials are given by $f(x) = x^4 - 2x^3 + 5$ and $g(x) = x^3 + 7x^2 - 3$.
Expand and simplify $2f(x) - g(x)$.

Solution

$$\begin{aligned}2f(x) - g(x) &= 2(x^4 - 2x^3 + 5) - (x^3 + 7x^2 - 3)\\ &= 2x^4 - 4x^3 + 10 - x^3 - 7x^2 + 3\\ &= 2x^4 - 5x^3 - 7x^2 + 13\end{aligned}$$

Example 9

Polynomials are given by $p(x) = x^3 - 2$ and $q(x) = 2x^3 + 3x - 5$.

Expand and simplify $p(x)q(x)$.

Solution

$$\begin{aligned}p(x)q(x) &= (x^3 - 2)(2x^3 + 3x - 5)\\ &= 2x^6 + 3x^4 - 5x^3 - 4x^3 - 6x + 10\\ &= 2x^6 + 3x^4 - 9x^3 - 6x + 10\end{aligned}$$

×	$2x^3$	$3x$	-5
x^3	$2x^6$	$3x^4$	$-5x^3$
-2	$-4x^3$	$-6x$	10

Exercise D (answers p 185)

1 Expand and simplify $2(x^3 + 3x - 1) - x(x^2 + x - 5)$.

2 Work out the value of each polynomial when $x = 2$.

(a) $x^5 - x^3 + 6$ **(b)** $4x^3 + 5x^2 - 9$ **(c)** $2x^5 - 4x^3 - 5x^2 - 7x + 2$

3 Expand and simplify each of these.

(a) $(x + 2)(x^4 + 3x + 1)$ **(b)** $(x^2 + 6)^2$

(c) $(x^2 + 3x - 1)(2x^3 + x + 5)$ **(d)** $(1 - x)(x^3 - 1) + 6x$

(e) $(3x^2 - x)(x^2 + 3) + x^2$ **(f)** $(x^2 - 1)^3$

(g) $(x + 1)(x + 4)(x - 2)(x - 3)$ **(h)** $(1 - 2x)(x + 6)(3x - 2) + x(2x + 1)$

4 A polynomial is given by $f(x) = 2x^4 - x^3 - 2x + 6$.
Evaluate each of these.

(a) f(1) **(b)** f(0) **(c)** f(2) **(d)** f(–1) **(e)** f(–2)

5 A polynomial is given by $g(x) = x^3 + 2x^2 - 25x - 50$.

(a) Evaluate g(5), g(–5) and g(–2).

(b) Hence write down the coordinates of the points where the graph of $y = g(x)$ crosses the x-axis.

6 Polynomials are given by $p(x) = x^2 + 1$ and $q(x) = x^5 - 2x^2 - 2$.
Expand and simplify each of these.

(a) $2p(x) + q(x)$ **(b)** $3p(x) - q(x)$ **(c)** $p(x)q(x)$ **(d)** $(q(x))^2$

7 A polynomial is given by $f(x) = -2x^3 - 3x^2 + 3x + 2$.

(a) Evaluate each of these.

(i) f(0) **(ii)** f(1) **(iii)** f(2) **(iv)** $f(-\frac{1}{2})$ **(v)** f(–2)

(b) Sketch the graph of $y = f(x)$.

8 A function is given by $f(x) = (2x - 3)(x + 4)^2$.

(a) Write the function in the form $f(x) = ax^3 + bx^2 + cx + d$.

(b) Sketch the graph of $y = f(x)$.

9 A function is given by $g(x) = 2x^3 - 7x^2 + 3x$.

(a) Write $g(x)$ as the product of three linear factors.

(b) Solve the equation $g(x) = 0$.

10 A polynomial is given by $p(x) = 2x^4 + 3x + c$.
If p(1) = 10, find the value of c.

11 A polynomial is given by $q(x) = x^4 - x^3 + ax + b$.
q(0) = –5 and q(2) = 7.
Find the values of a and b.

Key points

- To find the equation of a curve after a translation of $\begin{bmatrix} p \\ q \end{bmatrix}$, replace x with $(x - p)$ and replace y with $(y - q)$. (p 90)
- The graph of a cubic polynomial meets the x-axis one, two or three times, depending on its shape and position. (p 92)
- The x-coordinates of the points where the graph of $y = \text{p}(x)$ meets the x-axis can be found by solving the equation $\text{p}(x) = 0$. (p 93)
- A **polynomial** is an expression that can be written in the form $a + bx + cx^2 + dx^3 + ex^4 + fx^5 + \ldots$, where $a, b, c, d, e, f, \ldots$ are constants. (p 96)

Mixed questions (answers p 185)

1 The graph of $y = x^3$ is translated by $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$.

(a) Which is the correct equation for the new graph?

A $y = (x + 4)^3 + 1$ **B** $y = (x + 4)^3 - 1$

C $y = (x - 4)^3 + 1$ **D** $y = (x - 4)^3 - 1$

(b) Write the correct equation in the form $y = ax^3 + bx^2 + cx + d$.

2 A polynomial is given by $\text{f}(x) = (2x + 1)(x + 2)(x - 3)$.

(a) Evaluate f(0).

(b) Sketch the graph of $y = \text{f}(x)$, showing clearly where the graph crosses both axes.

(c) Write the polynomial in the form $\text{f}(x) = ax^3 + bx^2 + cx + d$.

3 A function is defined as $\text{g}(x) = (x + 3)(x - 2)^2$.

(a) Evaluate g(0).

(b) Solve the equation $\text{g}(x) = 0$.

(c) Sketch the graph of $y = \text{g}(x)$, showing clearly where the graph meets both axes.

4 Polynomials are given by $\text{p}(x) = 3x^2 - 2$ and $\text{q}(x) = x^3 - x^2 + 3$.

(a) Evaluate each of these.

(i) p(0) **(ii)** q(3) **(iii)** q(–2)

(b) Expand and simplify each of these.

(i) $2\text{p}(x) - 3\text{q}(x)$ **(ii)** $\text{p}(x)\text{q}(x)$ **(iii)** $(\text{p}(x))^2$

5 A polynomial is given by $\text{p}(x) = 2x^3 - 18x$.

(a) Express $\text{p}(x)$ as a product of linear factors.

(b) Sketch the graph of $y = \text{p}(x)$, showing clearly where the graph crosses both axes.

***6** A function is defined as $f(x) = x^3 + 3x^2 + 2x$.

(a) Factorise $f(x)$ completely.

(b) Hence show that the value of $f(a)$ is a multiple of 3 for any integer a.

Test yourself (answers p 186)

None of these questions requires a calculator.

1 The graph of $y = x^3 + x$ is translated by $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

(a) Find the equation of the new graph in the form $y = ax^3 + bx^2 + cx + d$.

(b) Where does this graph cross the y-axis?

2 Match up each equation below with a sketch of its graph.

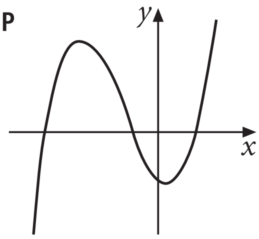
(a) $y = (3x + 2)(x - 1)(x + 3)$

(b) $y = (1 - 2x)(x - 6)(x + 1)$

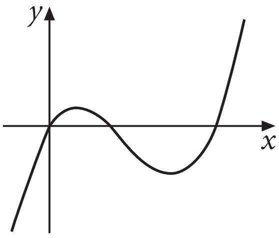
(c) $y = x^3 - 6x^2 + 7x$

(d) $y = (x - 2)(x^2 + 3x + 3)$

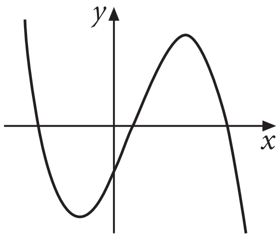
P



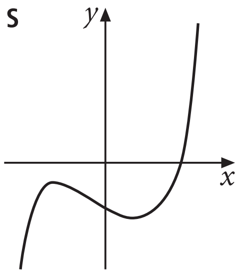
Q



R



S



3 A function is defined as $g(x) = (x + 2)(2x - 1)^2$.

(a) Evaluate $g(0)$ and $g(2)$.

(b) Solve the equation $g(x) = 0$.

(c) Sketch the graph of $y = g(x)$, showing clearly where the graph meets both axes.

(d) Write the function in the form $g(x) = ax^3 + bx^2 + cx + d$.

4 Polynomials are given by $p(x) = x^2 + 2x$ and $q(x) = x^3 - x^2 - 6x$.

(a) Expand and simplify each of these.

(i) $2p(x) + q(x)$ **(ii)** $p(x)q(x)$

(b) Solve the equation $q(x) = 0$.

(c) Find the values of x for which $p(x) = q(x)$.

***5** A function is defined as $f(x) = x^3 + 7x^2 + 6x$.

(a) Factorise $f(x)$ completely.

(b) Hence show that the value of $f(a)$ is even for any integer a.