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7 Polynomials

In this chapter you will learn how to

- evaluate and manipulate powers of the form kx^n , where n is a positive integer
- sketch cubic graphs and consider the effect of translations on their equations
- multiply out brackets to give a polynomial expression
- relate the roots of a polynomial to where its graph crosses the x -axis
- use function notation

A Indices: revision

Example 1

Evaluate $5x^3$ when $x = 2$.

Solution

$$\begin{aligned}\text{When } x = 2, \quad 5x^3 &= 5 \times 2^3 \\ &= 5 \times 8 \\ &= 40\end{aligned}$$

This is not the same as $(5 \times 2)^3$.

Example 2

Simplify $2x \times 3x^2$.

Solution

$$\begin{aligned}2x \times 3x^2 &= 2 \times x \times 3 \times x \times x \\ &= 2 \times 3 \times x \times x \times x \\ &= 6x^3\end{aligned}$$

Exercise A (answers p 181)

1 Evaluate these when $n = 3$.

- (a) $n^2 + n^3$ (b) $2n^2$ (c) $(2n)^2$ (d) $(n + 2)^3$ (e) $3(n + 1)^2$
(f) $3n^2 - 5n$ (g) $2n^5 - 3n$ (h) $\frac{1}{3}n^4 + 1$ (i) $2(n^3 - 7)$ (j) $(n - 1)^3 + n^3$

2 Evaluate these when $x = \frac{1}{2}$.

- (a) $6x^2$ (b) $(6x)^2$ (c) $5x(2x - 1)$ (d) $(x - 1)^3$

3 Evaluate these when $a = -1$.

- (a) $10a^4$ (b) a^9 (c) $5a^5$ (d) $4(a + 3)^3$

4 Simplify each of these.

- (a) $5x \times x^2$ (b) $4x^2 \times 3x$ (c) $2x^2 \times 5x^3$ (d) $\frac{1}{4}x^3 \times 8x$
(e) $5x^2 \times \frac{1}{3}x$ (f) $(2x)^3$ (g) $(3x^2)^2$ (h) $(\frac{1}{2}x^3)^2$

B Cubic graphs (answers p 181)

- B1** Use a graph plotter on a computer or graphic calculator to draw some graphs of the form $y = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants. Include some where the value of a is negative. What do you notice about the shapes of your graphs?

- B2** Use a graph plotter to draw each graph below. How many times does each one cross or touch the x -axis?

$$y = x^3 + 3x^2 + 4x + 5 \quad y = x^3 - 2x^2 - x + 2 \quad y = x^3 - x^2 - x + 1$$

- B3** Use a graph plotter to draw graphs of the three equations below.

$$y = x^3 \quad y = x^3 - x \quad y = x^3 + x$$

Describe the shape of each graph as fully as you can. Make sketches that show clearly the shape of each graph for $-1 \leq x \leq 1$.

- B4** For each equation below

- (i) make a sketch to show what you think the graph will look like
(ii) check by graphing on a graph plotter

(a) $y = -x^3$ (b) $y = x^3 + 2$ (c) $y = -x^3 - 5$

- B5** (a) Make a sketch of $y = x^3$.

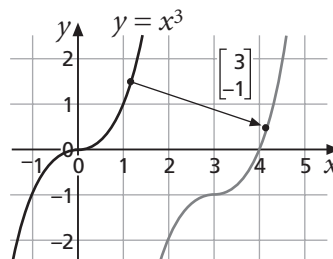
(b) Sketch what you think the graph of $y = (x - 2)^3$ will look like.

(c) Check on a graph plotter.

- B6** For each equation below, sketch what you think the graph will look like and then check on a graph plotter.

(a) $y = (x + 4)^3$ (b) $y = (x - 2)^3 + 5$ (c) $y = (x + 6)^3 - 7$

- B7** The graph of $y = x^3$ is translated by $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ as shown in the diagram.



- (a) Which do you think is the correct equation for the new graph?

A $y = (x - 3)^3 + 1$ B $y = (x - 3)^3 - 1$
C $y = (x + 3)^3 + 1$ D $y = (x + 3)^3 - 1$

(b) Check on a graph plotter.

(c) Work out where the new graph crosses the y -axis.

- B8** The graph of $y = x^3$ is translated by $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$.

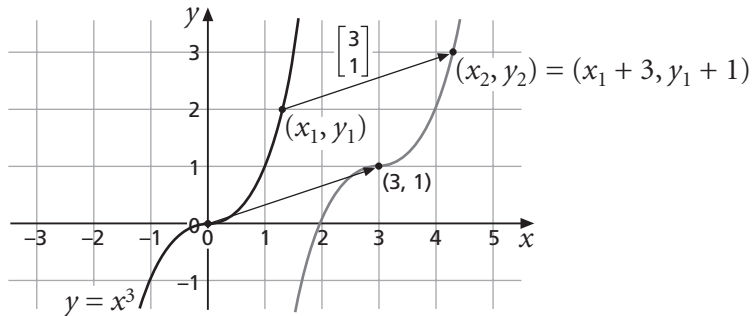
(a) Write down what you think is the correct equation for the translated graph.

(b) Work out where the translated graph crosses the y -axis.

We can work algebraically when translating curves.

In the diagram below, the graph of $y = x^3$ has been translated 3 units to the right and 1 unit up.

Using vector notation this translation is represented by $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$.



Let (x_1, y_1) be a point on $y = x^3$ and let (x_2, y_2) be its image on the translated curve.

Then $(x_2, y_2) = (x_1 + 3, y_1 + 1)$ giving

$$\begin{aligned} x_1 &= x_2 - 3 && \text{from rearranging } x_2 = x_1 + 3 \\ \text{and } y_1 &= y_2 - 1 && \text{from rearranging } y_2 = y_1 + 1 \end{aligned}$$

We know that $y_1 = x_1^3$, so it must be true that

$$y_2 - 1 = (x_2 - 3)^3$$

and so $y - 1 = (x - 3)^3$ is the equation of the transformed curve.

This can be written as $y = (x - 3)^3 + 1$.

So to find the equation of the curve that is the result of translating $y = x^3$ by $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ replace x by $(x - 3)$ and y by $(y - 1)$.

B9 Which translation will transform the curve $y = x^3$ to $y = (x - 5)^3 + 3$?

B10 The graph of $y = x^3 + x$ is translated by $\begin{bmatrix} 2 \\ -9 \end{bmatrix}$.

(a) Which do you think is the correct equation for the new graph?

- A** $y = (x + 2)^3 + (x + 2) + 9$ **B** $y = (x + 2)^3 + (x + 2) - 9$
C $y = (x - 2)^3 + (x - 2) + 9$ **D** $y = (x - 2)^3 + (x - 2) - 9$

(b) Check by graphing on a graph plotter.

(c) Work out where the new graph crosses the y -axis.

K To find the equation of any curve after it has been translated by $\begin{bmatrix} p \\ q \end{bmatrix}$, replace x by $(x - p)$ and replace y by $(y - q)$.

Example 3

Sketch the graph of $y = (x + 2)^3 + 5$.

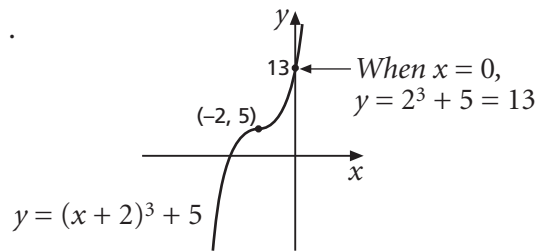
Solution

The equation can be written as $y - 5 = (x + 2)^3$.

This is $y = x^3$ with x replaced by $(x + 2)$ and y replaced by $(y - 5)$.

So the curve $y = x^3$ is translated by $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$.

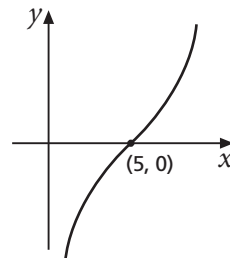
A sketch is shown on the right.



Example 4

The graph is a translation of $y = x^3 + x$ by $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$.

Find an equation for the graph.

**Solution**

The translation is $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ so replace x by $(x - 5)$, giving $y = (x - 5)^3 + (x - 5)$.

Exercise B (answers p 182)

- Which translation will transform the curve $y = x^3$ to each of these?
(a) $y - 2 = (x - 1)^3$ (b) $y + 5 = (x - 3)^3$ (c) $y - 1 = (x + 7)^3$
- Which translation will transform the curve $y = x^3$ to each of these?
(a) $y = (x - 4)^3 + 5$ (b) $y = (x + 2)^3 - 1$ (c) $y = (x + 6)^3 + 3$
- The curve $y = x^3$ is translated by $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$.
(a) Write an equation for the translated curve.
(b) Sketch the curve, showing where it crosses each axis.
- The curve $y = x^3$ is translated by $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$.
Write the equation of the translated curve in the form $y = (x + a)^3 + b$.
- Find the equation of the curve $y = x^3 + x$ after a translation of $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

6 The curve $y = x^3 + x$ is translated by $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

(a) Write the equation for the translated curve in the form $y = (x + p)^3 + x + q$.

(b) Work out where the curve crosses the y -axis.

7 The curve $y = x^3 - x$ is translated by $\begin{bmatrix} -3 \\ -2 \end{bmatrix}$.

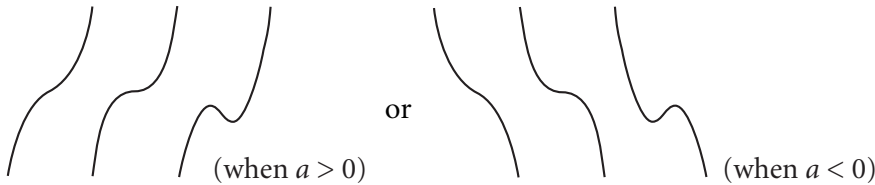
Write the equation for the translated curve in the form $y = (x + p)^3 - x + q$.

C Further graphs and manipulation (answers p 183)

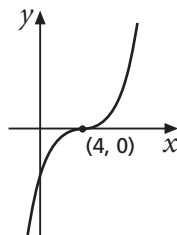
K

A **cubic** expression is one that can be written in the form $ax^3 + bx^2 + cx + d$ ($a \neq 0$), where a, b, c and d are constants.

Graphs of the form $y = ax^3 + bx^2 + cx + d$ have one of these basic shapes:



Here is a sketch of $y = (x - 4)^3$.



You can multiply out the brackets to write $y = (x - 4)^3$ in the form $y = ax^3 + bx^2 + cx + d$.

$$\begin{aligned} (x - 4)^3 &= (x - 4)(x - 4)(x - 4) \\ &= (x - 4)(x^2 - 4x - 4x + 16) \\ &= (x - 4)(x^2 - 8x + 16) \end{aligned}$$

$$\begin{aligned} &= x^3 - 8x^2 + 16x - 4x^2 + 32x - 64 \\ &= x^3 - 12x^2 + 48x - 64 \end{aligned}$$

×	x	-4
x	x^2	$-4x$
-4	$-4x$	16

×	x^2	$-8x$	16
x	x^3	$-8x^2$	$16x$
-4	$-4x^2$	$32x$	-64

So the equation of the graph can also be written as $y = x^3 - 12x^2 + 48x - 64$.

The constant multipliers in an expression such as $x^3 - 12x^2 + 48x - 64$ are called **coefficients**. In this example, the coefficient of x^2 is -12 .

C1 Write $y = (x + 3)^3$ in the form $y = ax^3 + bx^2 + cx + d$.

D C2 Each expression below is a product of three different linear expressions. Multiply out the brackets in each one.

- (a) $(x + 1)(x + 3)(x + 5)$ (b) $(2x + 1)(x + 2)(x - 3)$
 (c) $2x(x - 4)(3x + 1)$ (d) $(5x - 1)(3x - 1)(x + 4)$

- C3** (a) Solve the equation $(x + 3)(x + 2)(x - 1) = 0$.
 (b) What does this tell you about the graph of $y = (x + 3)(x + 2)(x - 1)$?
 (c) Where will the graph cross the y -axis?
 (d) Sketch the graph of $y = (x + 3)(x + 2)(x - 1)$.
 (e) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

- C4** (a) Where will the graph of $y = x(x + 1)(x - 4)$ cross the x -axis?
 (b) Sketch the graph of $y = x(x + 1)(x - 4)$.
 (c) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

- C5** (a) (i) Where will the graph of $y = (2x - 1)(x - 3)(x + 1)$ cross the x -axis?
 (ii) Where will it cross the y -axis?
 (b) Sketch the graph of $y = (2x - 1)(x - 3)(x + 1)$.
 (c) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

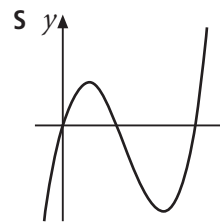
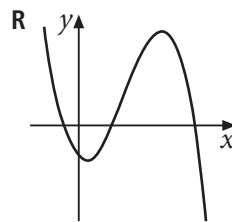
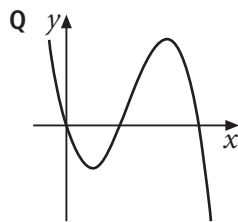
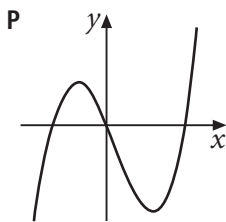
- D C6** (a) Solve the equation $(x + 3)(x - 1)^2 = 0$.
 (b) What does this tell you about the graph of $y = (x + 3)(x - 1)^2$?
 (c) Where will the graph cross the y -axis?
 (d) Sketch the graph of $y = (x + 3)(x - 1)^2$.
 (e) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

- C7** (a) Show that $(x + 3)^3 + 1$ is equivalent to $x^3 + 9x^2 + 27x + 28$.
 (b) Sketch the graph of $y = x^3 + 9x^2 + 27x + 28$.

- D C8** (a) Copy and complete the statement $3x^3 - x^2 - 4x = x(\quad)$.
 (b) Factorise $3x^3 - x^2 - 4x$ completely.
 (c) Sketch the graph of $y = 3x^3 - x^2 - 4x$.

- C9** (a) Solve the equation $x^2 - 2x - 4 = 0$.
 (b) Hence solve $x^3 - 2x^2 - 4x = 0$ and sketch the graph of $y = x^3 - 2x^2 - 4x$.

- C10** (a) Show that $x(x - 5)(2 - x)$ is equivalent to $-x^3 + 7x^2 - 10x$.
 (b) Decide which of the graphs below is a sketch of $y = -x^3 + 7x^2 - 10x$.



K

The product of three linear expressions can always be written in the form $ax^3 + bx^2 + cx + d$.

Example 5

Sketch the graph of $y = (2x - 1)(x - 3)(x + 2)$.

Solution

The expression is the product of three linear factors so the graph is a cubic shape.

The graph cuts the x -axis when $(2x - 1)(x - 3)(x + 2) = 0$.

The equation has three solutions:

$$2x - 1 = 0 \text{ gives } x = \frac{1}{2},$$

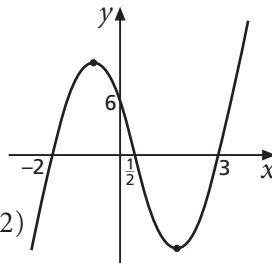
$$x - 3 = 0 \text{ gives } x = 3,$$

and $x + 2 = 0$ gives $x = -2$.

When $x = 0$, $y = -1 \times -3 \times 2 = 6$.

Hence, a sketch of the graph is

$$y = (2x - 1)(x - 3)(x + 2)$$



In Core 2, you will learn how to find the coordinates of the points marked with dots. Concentrate on the x - and y -intercepts for now.

Example 6

Write $(2x - 1)(x - 3)(x + 2)$ in the form $ax^3 + bx^2 + cx + d$.

Solution

$$\begin{aligned} & (2x - 1)(x - 3)(x + 2) \\ = & (2x - 1)(x^2 + 2x - 3x - 6) \end{aligned}$$

x	x	2
x	x^2	$2x$
-3	$-3x$	-6

$$\begin{aligned} = & (2x - 1)(x^2 - x - 6) \\ = & 2x^3 - 2x^2 - 12x - x^2 + x + 6 \\ = & 2x^3 - 3x^2 - 11x + 6 \end{aligned}$$

x	x^2	$-x$	-6
$2x$	$2x^3$	$-2x^2$	$-12x$
-1	$-x^2$	x	6

Example 7

Solve the equation $2x^3 - 10x^2 + 12x = 0$.

Solution

$2x$ is a factor of each term.

The quadratic factorises.

$$2x^3 - 10x^2 + 12x = 0$$

$$2x(x^2 - 5x + 6) = 0$$

$$2x(x - 2)(x - 3) = 0$$

So the solutions are $x = 0$, $x = 2$ and $x = 3$.

Exercise C (answers p 184)

1 For each equation below

(i) sketch a graph, showing clearly where it crosses the x - and y -axes

(ii) write the equation in the form $y = ax^3 + bx^2 + cx + d$

(a) $y = (x + 1)(x + 3)(x + 4)$

(b) $y = (x + 2)^3$

(c) $y = (x + 4)(x - 1)(x + 2)$

(d) $y = x(x + 3)(x - 2)$

(e) $y = (x - 5)^3$

(f) $y = (2x + 1)(x - 2)(x + 2)$

(g) $y = \frac{1}{2}x(2x + 3)(x - 2)$

(h) $y = (x + 4)^3 + 1$

2 (a) Solve the equation $(x + 1)(x - 3)^2 = 0$.

(b) Sketch the graph of $y = (x + 1)(x - 3)^2$, showing clearly where it meets both axes.

(c) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

3 (a) Copy and complete the statement $3x^3 - 3x^2 - 18x = 3x(\quad)$.

(b) Factorise $3x^3 - 3x^2 - 18x$ completely.

(c) Sketch the graph of $y = 3x^3 - 3x^2 - 18x$, showing clearly where it crosses both axes.

4 (a) Factorise $x^3 + 5x^2 - 6x$ completely.

(b) Solve the equation $x^3 + 5x^2 - 6x = 0$.

5 Sketch the graph of $y = 2x^3 + 14x^2 + 24x$, showing clearly where it crosses both axes.

6 (a) Find the value of $y = (2x - 3)(x + 1)(3 - x)$ when $x = 1$.

(b) Sketch the graph of $y = (2x - 3)(x + 1)(3 - x)$.

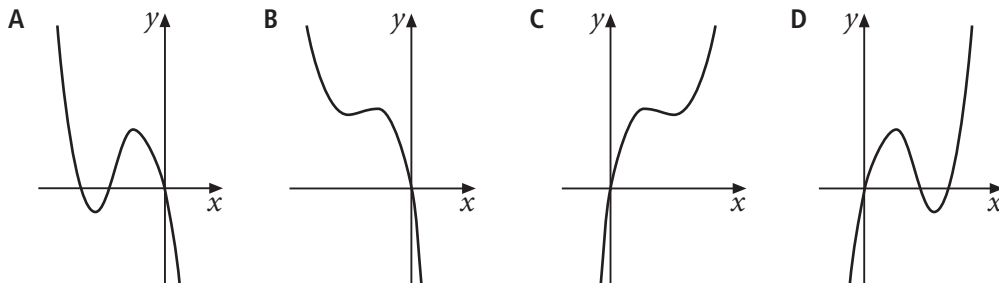
(c) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

7 Solve the equation $5x^3 - 20x^2 + 20x = 0$.

8 (a) Solve the equation $x^3 - 10x^2 + 22x = 0$, giving each solution in surd form.

(b) Sketch the graph of $y = x^3 - 10x^2 + 22x$, showing where it crosses both axes.

9 Which of the graphs below is a sketch of $y = x^3 - 4x^2 + 5x$?



10 Sketch the graph of $y = (2x + 3)^2(x - 1)$, showing clearly where it meets both axes.

D Polynomial functions

K A **polynomial** is an expression that can be written in the form $a + bx + cx^2 + dx^3 + ex^4 + fx^5 + \dots$, where a, b, c, d, e, f, \dots are constants.

Some examples of polynomials are $7x^4 + x - 10$ and $6 + 5x^2 - x^3 - 5x^7$.

Expressions such as $\sqrt{x} + 3x^2 - 5$ and $\frac{1}{x} + 7x$ are not polynomials as they involve terms in x that cannot be written in the form x^n where n is a positive integer (\sqrt{x} and $\frac{1}{x}$).

The **degree** of a polynomial is the value of its highest index.

For example, $6 + 5x^2 - x^3 - 5x^7$ is a polynomial of degree 7.

The sum, difference or product of two polynomials is also a polynomial.

For example,

$(x^4 - 5x)(3x^3 + x - 10)$	\times	$3x^3$	x	-10
$= 3x^7 + x^5 - 10x^4 - 15x^4 - 5x^2 + 50x$	x^4	$3x^7$	x^5	$-10x^4$
$= 3x^7 + x^5 - 25x^4 - 5x^2 + 50x$	$-5x$	$-15x^4$	$-5x^2$	$50x$

Function notation is useful when dealing with polynomials.

For example, if a polynomial function is defined by $f(x) = x^3 + 4x^2 - 1$, then $f(3)$ is the value of the polynomial at $x = 3$.

$$\begin{aligned} \text{So } f(3) &= 3^3 + 4 \times 3^2 - 1 \\ &= 27 + 36 - 1 \\ &= 62 \end{aligned}$$

Example 8

Polynomials are given by $f(x) = x^4 - 2x^3 + 5$ and $g(x) = x^3 + 7x^2 - 3$.
Expand and simplify $2f(x) - g(x)$.

Solution

$$\begin{aligned} 2f(x) - g(x) &= 2(x^4 - 2x^3 + 5) - (x^3 + 7x^2 - 3) \\ &= 2x^4 - 4x^3 + 10 - x^3 - 7x^2 + 3 \\ &= 2x^4 - 5x^3 - 7x^2 + 13 \end{aligned}$$

Example 9

Polynomials are given by $p(x) = x^3 - 2$ and $q(x) = 2x^3 + 3x - 5$.
Expand and simplify $p(x)q(x)$.

Solution

$p(x)q(x) = (x^3 - 2)(2x^3 + 3x - 5)$	\times	$2x^3$	$3x$	-5
$= 2x^6 + 3x^4 - 5x^3 - 4x^3 - 6x + 10$	x^3	$2x^6$	$3x^4$	$-5x^3$
$= 2x^6 + 3x^4 - 9x^3 - 6x + 10$	-2	$-4x^3$	$-6x$	10

Exercise D (answers p 185)

- 1** Expand and simplify $2(x^3 + 3x - 1) - x(x^2 + x - 5)$.
- 2** Work out the value of each polynomial when $x = 2$.
(a) $x^5 - x^3 + 6$ (b) $4x^3 + 5x^2 - 9$ (c) $2x^5 - 4x^3 - 5x^2 - 7x + 2$
- 3** Expand and simplify each of these.
(a) $(x + 2)(x^4 + 3x + 1)$ (b) $(x^2 + 6)^2$
(c) $(x^2 + 3x - 1)(2x^3 + x + 5)$ (d) $(1 - x)(x^3 - 1) + 6x$
(e) $(3x^2 - x)(x^2 + 3) + x^2$ (f) $(x^2 - 1)^3$
(g) $(x + 1)(x + 4)(x - 2)(x - 3)$ (h) $(1 - 2x)(x + 6)(3x - 2) + x(2x + 1)$
- 4** A polynomial is given by $f(x) = 2x^4 - x^3 - 2x + 6$.
Evaluate each of these.
(a) $f(1)$ (b) $f(0)$ (c) $f(2)$ (d) $f(-1)$ (e) $f(-2)$
- 5** A polynomial is given by $g(x) = x^3 + 2x^2 - 25x - 50$.
(a) Evaluate $g(5)$, $g(-5)$ and $g(-2)$.
(b) Hence write down the coordinates of the points where the graph of $y = g(x)$ crosses the x -axis.
- 6** Polynomials are given by $p(x) = x^2 + 1$ and $q(x) = x^5 - 2x^2 - 2$.
Expand and simplify each of these.
(a) $2p(x) + q(x)$ (b) $3p(x) - q(x)$ (c) $p(x)q(x)$ (d) $(q(x))^2$
- 7** A polynomial is given by $f(x) = -2x^3 - 3x^2 + 3x + 2$.
(a) Evaluate each of these.
(i) $f(0)$ (ii) $f(1)$ (iii) $f(2)$ (iv) $f(-\frac{1}{2})$ (v) $f(-2)$
(b) Sketch the graph of $y = f(x)$.
- 8** A function is given by $f(x) = (2x - 3)(x + 4)^2$.
(a) Write the function in the form $f(x) = ax^3 + bx^2 + cx + d$.
(b) Sketch the graph of $y = f(x)$.
- 9** A function is given by $g(x) = 2x^3 - 7x^2 + 3x$.
(a) Write $g(x)$ as the product of three linear factors.
(b) Solve the equation $g(x) = 0$.
- 10** A polynomial is given by $p(x) = 2x^4 + 3x + c$.
If $p(1) = 10$, find the value of c .
- 11** A polynomial is given by $q(x) = x^4 - x^3 + ax + b$.
 $q(0) = -5$ and $q(2) = 7$.
Find the values of a and b .

Key points

- To find the equation of a curve after a translation of $\begin{bmatrix} p \\ q \end{bmatrix}$, replace x with $(x - p)$ and replace y with $(y - q)$. (p 90)
- The graph of a cubic polynomial meets the x -axis one, two or three times, depending on its shape and position. (p 92)
- The x -coordinates of the points where the graph of $y = p(x)$ meets the x -axis can be found by solving the equation $p(x) = 0$. (p 93)
- A **polynomial** is an expression that can be written in the form $a + bx + cx^2 + dx^3 + ex^4 + fx^5 + \dots$, where a, b, c, d, e, f, \dots are constants. (p 96)

Mixed questions (answers p 185)

- 1 The graph of $y = x^3$ is translated by $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$.
- (a) Which is the correct equation for the new graph?
- A $y = (x + 4)^3 + 1$ B $y = (x + 4)^3 - 1$
C $y = (x - 4)^3 + 1$ D $y = (x - 4)^3 - 1$
- (b) Write the correct equation in the form $y = ax^3 + bx^2 + cx + d$.
- 2 A polynomial is given by $f(x) = (2x + 1)(x + 2)(x - 3)$.
- (a) Evaluate $f(0)$.
- (b) Sketch the graph of $y = f(x)$, showing clearly where the graph crosses both axes.
- (c) Write the polynomial in the form $f(x) = ax^3 + bx^2 + cx + d$.
- 3 A function is defined as $g(x) = (x + 3)(x - 2)^2$.
- (a) Evaluate $g(0)$.
- (b) Solve the equation $g(x) = 0$.
- (c) Sketch the graph of $y = g(x)$, showing clearly where the graph meets both axes.
- 4 Polynomials are given by $p(x) = 3x^2 - 2$ and $q(x) = x^3 - x^2 + 3$.
- (a) Evaluate each of these.
- (i) $p(0)$ (ii) $q(3)$ (iii) $q(-2)$
- (b) Expand and simplify each of these.
- (i) $2p(x) - 3q(x)$ (ii) $p(x)q(x)$ (iii) $(p(x))^2$
- 5 A polynomial is given by $p(x) = 2x^3 - 18x$.
- (a) Express $p(x)$ as a product of linear factors.
- (b) Sketch the graph of $y = p(x)$, showing clearly where the graph crosses both axes.

*6 A function is defined as $f(x) = x^3 + 3x^2 + 2x$.

- (a) Factorise $f(x)$ completely.
 (b) Hence show that the value of $f(a)$ is a multiple of 3 for any integer a .

Test yourself (answers p 186)

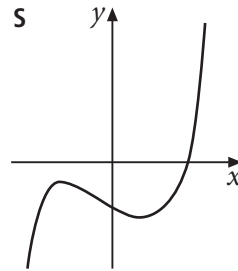
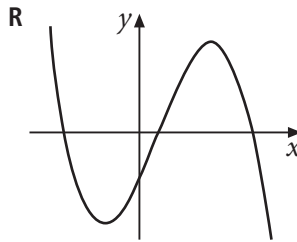
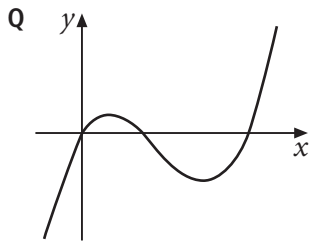
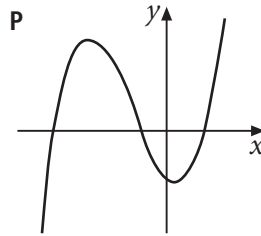
None of these questions requires a calculator.

1 The graph of $y = x^3 + x$ is translated by $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

- (a) Find the equation of the new graph in the form $y = ax^3 + bx^2 + cx + d$.
 (b) Where does this graph cross the y -axis?

2 Match up each equation below with a sketch of its graph.

- (a) $y = (3x + 2)(x - 1)(x + 3)$
 (b) $y = (1 - 2x)(x - 6)(x + 1)$
 (c) $y = x^3 - 6x^2 + 7x$
 (d) $y = (x - 2)(x^2 + 3x + 3)$



3 A function is defined as $g(x) = (x + 2)(2x - 1)^2$.

- (a) Evaluate $g(0)$ and $g(2)$.
 (b) Solve the equation $g(x) = 0$.
 (c) Sketch the graph of $y = g(x)$, showing clearly where the graph meets both axes.
 (d) Write the function in the form $g(x) = ax^3 + bx^2 + cx + d$.

4 Polynomials are given by $p(x) = x^2 + 2x$ and $q(x) = x^3 - x^2 - 6x$.

- (a) Expand and simplify each of these.
 (i) $2p(x) + q(x)$ (ii) $p(x)q(x)$
 (b) Solve the equation $q(x) = 0$.
 (c) Find the values of x for which $p(x) = q(x)$.

*5 A function is defined as $f(x) = x^3 + 7x^2 + 6x$.

- (a) Factorise $f(x)$ completely.
 (b) Hence show that the value of $f(a)$ is even for any integer a .