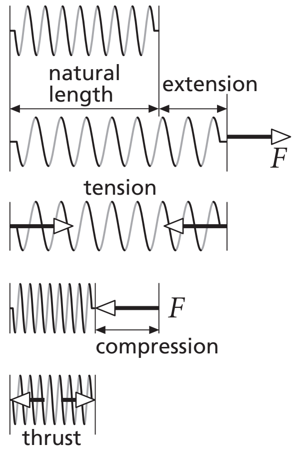
Mechanics 2 for AQA contents

7 Hooke's law

In this chapter you will learn how to

- use Hooke's law
- find the work done in stretching a spring or an elastic string
- solve problems involving elastic potential energy

A Elastic springs and strings (answers p 171)



An **elastic** spring is one which will stretch when a force is applied to it.
When there is no force applied, the length of the unstretched spring is its **natural length**.
The amount that it stretches under a force is the **extension**.
When the spring is stretched there is a tension in the spring which exerts equal and opposite forces at the ends.

If the force is applied in the opposite direction, it will cause the spring to compress. The amount by which its length has been reduced is the **compression**.
When the spring is compressed there is a thrust in the spring which exerts equal and opposite forces at the ends.

Experiments show that when a spring is stretched, the tension is directly proportional to the extension. When it is compressed the thrust is directly proportional to the compression. The tension or thrust depends on the natural length of the spring and how the spring is made.

The English scientist and architect Robert Hooke (1635–1703) discovered that the tension (or thrust) is proportional to the extension (or compression) of the spring. This fact is incorporated in a formula known as Hooke's law.

The tension (or thrust), T N, in a spring of natural length l m is related to the extension (or compression) x m by the formula

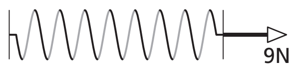
$$T = \frac{\lambda x}{l} \quad \text{(Hooke's law)}$$

λ is a constant for a particular spring and is known as its modulus of elasticity; it is measured in newtons.

Hooke's law also applies to elastic strings, but these can only be extended and not compressed.

A1 An elastic string of natural length 0.5 m has modulus of elasticity 15 N. Find the tension in the string when it is extended by 0.1 m.

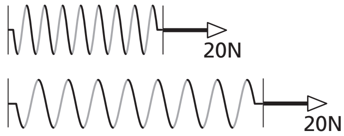
A2 An elastic spring of natural length 0.2 m extends by 0.05 m when a force of 9 N is applied to it.



(a) What is the magnitude of the tension in the spring?

(b) Find the modulus of elasticity of the spring.

A3 Two elastic springs both have modulus of elasticity 25 N. One spring has natural length 0.4 m and the other has natural length 0.8 m.
Both springs are fixed at one end and have a force of 20 N applied to the free end.

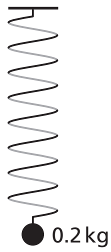


(a) Find the extension of the 0.4 m spring.

(b) Find the extension of the 0.8 m spring.

(c) Explain why the springs do not have the same extension.

A4 A spring of negligible weight is fixed at one end and has a particle of mass 0.2 kg hung on the other end. The spring hangs in equilibrium.



(a) Draw a force diagram for the particle.

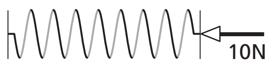
(b) Find the magnitude of the tension in the spring.

(c) Given that the extension of the spring is equal to its natural length, use Hooke's law to find the modulus of elasticity of the spring. Comment on your answer.

The modulus of elasticity of a spring is equal to the force needed to double the spring's length. When the spring's extension is equal to its natural length, then the tension is equal to the modulus of elasticity. The modulus of elasticity depends on the material of the spring and on its geometric properties.

Hooke's law only applies up to a point, known as the elastic limit of the spring. If the spring is stretched further than this point, it will not return to its natural length when released, and Hooke's law no longer applies.
Also a spring can only be compressed up to a certain point.
None of the springs considered in this chapter will reach these limits.

A5 A spring of natural length 0.4 m is compressed by a force of 10 N. The modulus of elasticity of the spring is 40 N.



(a) Find the distance by which the spring is compressed.

(b) What is the length of the spring under the action of the force?

Example 1

An elastic spring of natural length 1.4 m is stretched to a length of 2.2 m by a force of 16 N. Find the modulus of elasticity of the spring.

Solution

Remember to use the extension, not the stretched length, in the Hooke's law equation.

Use $T = \dfrac{\lambda x}{l}$.

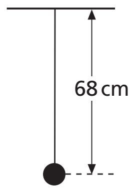
$$16 = \frac{(2.2 - 1.4)\lambda}{1.4} = \frac{0.8\lambda}{1.4}$$

$$\Rightarrow \lambda = \frac{16 \times 1.4}{0.8} = 28$$

The modulus of elasticity of the spring is 28 N.

Example 2

An elastic string with modulus of elasticity 7 N is fixed at one end.
A particle of mass 0.5 kg is suspended from the other end and hangs in equilibrium.
The length of the stretched string is 68 cm.
Find the natural length and extension of the string.



Solution

Draw a force diagram for the particle.
Note that in calculations all lengths should be in metres and masses in kilograms.

The particle is in equilibrium. $\quad T = 0.5g = 4.9$

Use $T = \dfrac{\lambda x}{l}$. $\quad 4.9 = \dfrac{7x}{l}$

$\Rightarrow \quad x = \dfrac{4.9l}{7} = 0.7l$

The stretched length is $l + x$. $\quad l + x = 0.68$

Substitute for x. $\quad \Rightarrow \quad 1.7l = 0.68 \Rightarrow l = 0.4$

$\Rightarrow \quad x = 0.7 \times 0.4 = 0.28$

The natural length of the string is 0.4 m and its extension is 0.28 m.

Exercise A (answers p 171)

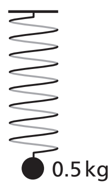
1 An elastic string of natural length 0.8 m has modulus of elasticity 24 N.
Find the tension in the string when it is extended by

(a) 0.1 m **(b)** 0.2 m **(c)** 0.4 m

2 An elastic spring of natural length 0.2 m has modulus of elasticity 40 N.
Find its extension when the tension in the spring is

(a) 6 N **(b)** 8 N **(c)** 12 N

3 A light spring extends by 0.4 m when a particle of mass 0.5 kg is hung on one end.
The natural length of the spring is 1.2 m.

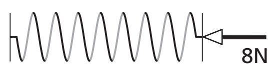


(a) Find the tension in the spring.

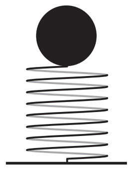
(b) Find the modulus of elasticity of the spring.

4 An elastic string of natural length 0.9 m is stretched to a length of 1.5 m by a force of 21 N.
Find the modulus of elasticity of the string.

5 A light spring of natural length 0.4 m and modulus of elasticity 20 N is compressed by a force of 8 N.
Find the compressed length of the spring.



6 A light spring has modulus of elasticity 14 N and natural length 0.05 m. A particle of mass 0.2 kg is placed on top of the spring as shown.

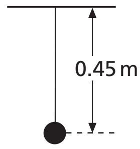


(a) Find the magnitude of the thrust in the spring.

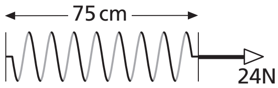
(b) Find the amount by which the spring is compressed.

(c) What is the compressed length of the spring?

7 An elastic string of natural length 0.3 m is fixed at one end and has a particle of mass 0.25 kg attached at the other end.
The particle hangs in equilibrium 0.45 m below the fixed end.
Show that the modulus of elasticity of the string is 4.9 N.



8 A spring has modulus of elasticity 16 N.
It is stretched to a length of 75 cm by a force of 24 N.
Find the natural length and extension of the spring.



9 A light spring with modulus of elasticity 7 N is fixed at one end and has a particle of mass 0.25 kg attached at the other end. The particle hangs in equilibrium and the length of the stretched spring is 108 cm.
Find the natural length and extension of the spring.

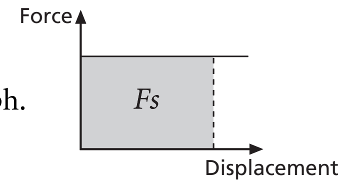
*10 A string is stretched to a length of 38 cm when a force of 15 N is applied. When the force is increased to 21 N, the string's length becomes 46 cm.
Find the natural length and modulus of elasticity of the string.

B Work done by a variable force (answers p 172)

In the last chapter we learned how to find the work done by a constant force.

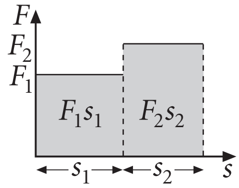
The graph shows the relationship between force and displacement for a constant force.

The work done by the force F is Fs, which is the area under the graph.



Now consider a force F_1 which acts on a body for a displacement s_1. Then it increases to F_2 and acts on the body for a displacement s_2.

The force–displacement graph is shown.



Work done by $F_1 = F_1 s_1$

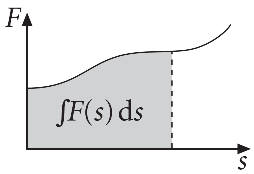
Work done by $F_2 = F_2 s_2$

The total work done by the forces is therefore $F_1 s_1 + F_2 s_2$, which is the area under the step graph.

This can be generalised to give an expression for the work done by any variable force.

K The work done by a variable force $F(s)$ is given by

$$\text{work done} = \int F(s)\, \mathrm{d}s$$

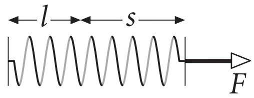


D

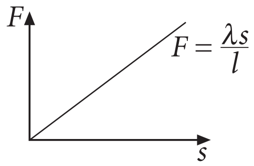
B1 An elastic spring is stretched by applying a force to one end.
Use Hooke's law to explain why the force is not constant as the spring stretches.

Imagine that a spring is being stretched, starting from its natural length l m.

When the extension is s m, the tension, and therefore the force F applied to the spring, is given by $F = \frac{\lambda s}{l}$.

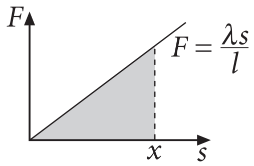


As the spring is stretched and s increases, the value of F increases, as shown by the graph, whose equation is $F = \frac{\lambda s}{l}$.



When the extension reaches x m, the work done in stretching the spring is

$$\int_0^x F\,\mathrm{d}s = \int_0^x \frac{\lambda s}{l}\,\mathrm{d}s = \left[\frac{\lambda s^2}{2l}\right]_0^x = \frac{\lambda x^2}{2l}$$



K

The work done, in joules, in stretching an elastic spring with modulus of elasticity λ N and natural length l m from its natural length to an extension x m is given by

$$\text{work done} = \frac{\lambda x^2}{2l}$$

B2 An elastic spring has natural length 0.5 m and modulus of elasticity 12 N.
Find the work done in extending the spring by 0.1 m.

B3 A spring has natural length 0.8 m and modulus of elasticity 20 N.
It is stretched from its natural length by 0.2 m.

(a) Find the work done in stretching the spring by 0.2 m.

(b) The spring is now stretched by a further 0.2 m.

(i) What is the total extension of the spring?

(ii) Find the total work done in stretching the spring to this length.

(iii) Find the work done in stretching the spring the further 0.2 m.

(c) Sketch a graph of force against extension for the spring and use it to explain why your answers to (a) and (b)(iii) were not the same.

As the spring is stretched, work is being done on it by the applied force which causes the spring to gain energy. This energy is stored in the spring as **elastic potential energy**. When the spring is allowed to contract, this elastic potential energy is released.

K

The elastic potential energy (e.p.e.), in joules, of a stretched spring with extension x m is equal to the work done in extending the spring from its natural length to this extension.

$$\text{e.p.e.} = \frac{\lambda x^2}{2l}$$

Note that, because of the x^2 term, the elastic potential energy of a spring is always positive, whether the spring has been extended or compressed.

Example 3

A spring of natural length 0.9 m is compressed to a length of 0.6 m.
The modulus of elasticity of the spring is 25 N.
Find the elastic potential energy stored in the spring.

Solution

The compressed length is 0.6 m, *so the spring has been compressed by* 0.3 m.

e.p.e. $= \frac{\lambda x^2}{2l} = \frac{25 \times 0.3^2}{2 \times 0.9} = 1.25\,\text{J}$

Example 4

An elastic string has natural length 1.2 m and modulus of elasticity 40 N.
Find the work done in stretching the string from a length of 1.35 m to 1.5 m.

Solution

Find the work done to stretch the string to 1.35 m, *an extension of* 0.15 m.

Work done $= \frac{\lambda x^2}{2l} = \frac{40 \times 0.15^2}{2 \times 1.2} = 0.375\,\text{J}$

Now find the work done to stretch the string from its natural length to 1.5 m,
an extension of 0.3 m.

Work done $= \frac{\lambda x^2}{2l} = \frac{40 \times 0.3^2}{2 \times 1.2} = 1.5\,\text{J}$

The work done in stretching from 1.35 m *to* 1.5 m *is the difference between these two amounts.*

Work done in stretching from 1.35 m to 1.5 m = 1.5 – 0.375 = 1.125 J

Exercise B (answers p 172)

1 An elastic string has modulus of elasticity 20 N and natural length 0.5 m.
Find the work done in stretching the string from its natural length by

(a) 0.1 m **(b)** 0.2 m **(c)** 0.4 m

2 An elastic string of natural length 0.75 m has modulus of elasticity 12 N.
Find the work done in stretching the string to double its natural length.

3 Two elastic springs both have modulus of elasticity 15 N.
One spring has natural length 0.3 m and the other has natural length 0.6 m.
They are both stretched by a force of 10 N.

(a) (i) Find the extension of the 0.3 m spring.

(ii) Find the elastic potential energy stored in the spring.

(b) (i) Find the extension of the 0.6 m spring.

(ii) Find the elastic potential energy stored in this spring.

4 An elastic spring of natural length 0.6 m is compressed to a length of 0.4 m.
The modulus of elasticity of the spring is 6 N.
Find the elastic potential energy stored in the spring.

5 An elastic string hangs vertically in equilibrium with a particle of mass 0.1 kg attached.
The natural length of the string is 1 m and its modulus of elasticity is 14 N.

(a) Find the extension of the string.

(b) Find the elastic potential energy stored in the string.

6 A spring has modulus of elasticity 10 N and natural length 24 cm.

(a) Find the work done to increase its length by 6 cm.

(b) Find the work done to increase its length by a further 6 cm.

7 A particle of mass 0.5 kg is suspended in equilibrium from a fixed point by an elastic string of natural length 0.8 m and modulus of elasticity 35 N.

(a) Find the elastic potential energy stored in the string.

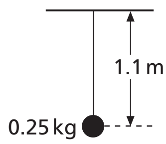
(b) Find the work done in pulling the particle down a further 0.1 m.

8 An elastic string is stretched to a length of 0.75 m by a force of 5 N.
The modulus of elasticity of the string is 20 N.

(a) Find the natural length and extension of the string.

(b) Find the elastic potential energy stored in the string.

9 An elastic string is fixed at one end and has a particle of mass 0.25 kg attached at the other end. The particle hangs in equilibrium 1.1 m below the fixed point. The modulus of elasticity of the string is 24.5 N.
Find the elastic potential energy stored in the spring.



***10** A spring is stretched to a length 0.6 m.
The work done in stretching it from this length to a length of 0.8 m is 2 J.
The modulus of elasticity of the spring is 10 N.
Find its natural length.

C Mechanical energy (answers p 172)

In the previous chapter we saw that the sum of the potential energy and the kinetic energy of a system remains constant if no external force other than gravity does work.

In the case of elastic strings and springs, elastic potential energy, another form of mechanical energy, is introduced. Here again the principle of conservation of mechanical energy can be applied if no external force other than gravity does work.

An elastic string is fixed at one end with a particle attached at the free end. The particle rests on a smooth horizontal surface and the string is slack.

A horizontal force is applied to the particle, causing the string to stretch.
The force has done work so the elastic potential energy of the string has increased.

D

C1 The force is now removed from the particle.

(a) Describe the subsequent motion and how the kinetic energy and elastic potential energy of the system change during the motion.

(b) Can the principle of conservation of mechanical energy be applied to this motion? Explain your answer.

(c) How would your answer to (b) differ if the surface had been rough rather than smooth?

(d) Describe the motion of the particle if it were resting on a smooth surface but attached to a spring rather than a string.

C2 A particle is attached to the end of a light spring and it hangs in equilibrium. The particle is pulled down a small distance and released.

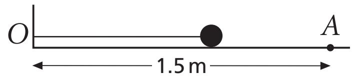
(a) Describe the subsequent motion and how the kinetic energy, gravitational potential energy and elastic potential energy of the system change during the motion.

(b) Can the principle of conservation of mechanical energy be applied to this motion? Explain your answer.

K

The total mechanical energy of a system remains constant if no external force other than gravity does work.
If the principle of conservation of mechanical energy applies, then the sum of the gravitational potential energy, elastic potential energy and kinetic energy of the system is constant.

Consider an elastic string of natural length 1 m and modulus of elasticity 20 N which is fixed at one end to point O and has a particle of mass 0.2 kg attached to the other end.
The particle lies on a smooth horizontal surface and is pulled to point A then released from rest.



C3 **(a)** When the particle is at A find

(i) the elastic potential energy **(ii)** the kinetic energy

(b) When the particle is 1.2 m from O find

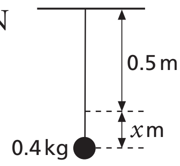
(i) the elastic potential energy **(ii)** the kinetic energy

The total mechanical energy is constant throughout the motion as there is no friction force acting. The elastic potential energy has its maximum value when the particle is released. As the string contracts back to its natural length this elastic potential energy is converted to kinetic energy.
As the motion is horizontal there is no change in gravitational potential energy.

C4 **(a)** At what point is the kinetic energy greatest?
What is the value of the elastic potential energy at this point?

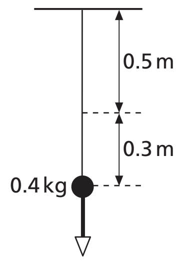
(b) Find the maximum speed of the particle.

Consider an elastic string of natural length 0.5 m and modulus of elasticity 19.6 N which is suspended from a fixed point with a particle of mass 0.4 kg attached to the free end.



C5 Find the length of the string when the particle hangs in equilibrium.

The particle is now pulled vertically downwards, so that the length of the string is 0.8 m, then released. This is beyond the equilibrium position of the string so the particle will start to move upwards as the string contracts on release. The total mechanical energy of the system is constant because, if air resistance can be ignored, the only external force acting is the weight of the particle.



C6 **(a)** Find the elastic potential energy when the particle is released.

(b) When the particle passes its equilibrium position,

(i) find the elastic potential energy

(ii) find the gravitational potential energy relative to the release point

(iii) hence find the kinetic energy

(c) When the string becomes slack,

(i) find the elastic potential energy

(ii) find the gravitational potential energy relative to the release point

(iii) hence find the kinetic energy

Once the string has become slack, the elastic potential energy is zero. The particle continues to move upwards, now acting as a projectile.

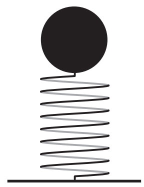
C7 **(a)** What is the kinetic energy of the particle when it reaches its highest point?

(b) Find the particle's gravitational potential energy relative to the release point when the particle reaches its highest point.

(c) What is the maximum height the particle reaches above its release point?

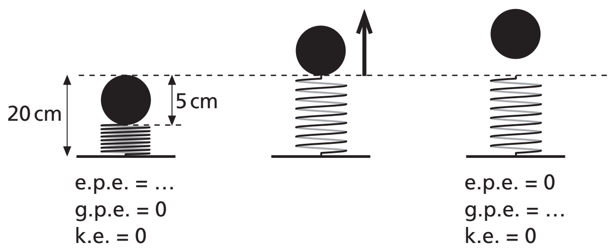
Example 5

A particle of mass 40 g is placed on top of a light spring of natural length 20 cm and modulus of elasticity 50 N resting on a horizontal surface.
The spring is compressed by 5 cm and released.
Find the maximum height above the surface reached by the particle.



Solution

All lengths must be converted to metres and masses to kilograms.
There are no external forces except gravity acting, so mechanical energy is conserved.



Initially the kinetic energy is zero, and it is also zero when the particle reaches its maximum height. So when the particle is at its maximum height all the initial elastic potential energy has been converted to gravitational potential energy.

Initial e.p.e. $= \dfrac{\lambda x^2}{2l} = \dfrac{50 \times 0.05^2}{2 \times 0.2} = 0.3125$

The particle rises h m from its initial position.

Gain in g.p.e. $= mgh = 0.04 \times 9.8h = 0.392h$

Gain in g.p.e. = loss in e.p.e. $\quad 0.392h = 0.3125$

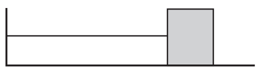
$\Rightarrow \quad h = 0.797$ to 3 s.f.

The particle was initially 0.15 m above the surface, so the maximum height reached above the surface is 0.947 m to 3 s.f.

If the principle of conservation of mechanical energy does not apply then the problem can be solved using the work–energy principle, as demonstrated in the following example.

Example 6

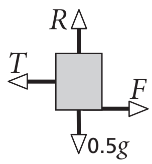
One end of an elastic string is fixed to a wall and the other end is attached to a box of mass 0.5 kg. The box rests on a rough horizontal surface as shown. The string has natural length 0.4 m and modulus of elasticity 80 N. The coefficient of friction between the box and the surface is 0.25. The box is pulled so that it is 0.6 m from the wall and released from rest.



Find the speed of the box when the string becomes slack.

Solution

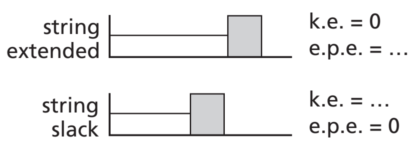
The motion is horizontal so the gravitational potential energy is constant.
The surface is rough so mechanical energy is not conserved.
Draw a force diagram for the box.



Resolve the forces vertically. $\quad R = 0.5g = 4.9$

The box is moving, so $F = \mu R$. $\quad F = 0.25 \times 4.9 = 1.225$

When the box is released, the extension of the string is 0.2 m.
It starts from rest so the initial kinetic energy is zero.
When the string becomes slack, the box has moved 0.2 m *and the elastic potential energy is zero.*
The friction force acts in the opposite direction to the direction of motion so it does negative work and the total mechanical energy has decreased.



Initial e.p.e. $= \dfrac{\lambda x^2}{2l} = \dfrac{80 \times 0.2^2}{2 \times 0.4} = 4$

Gain in k.e. $= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5 \times v^2 = 0.25v^2$

Work done by friction $= -1.225 \times 0.2 = -0.245$

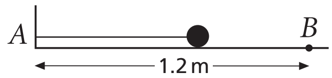
Work done = change in energy $\quad -0.245 = 0.25v^2 - 4$

$\Rightarrow \quad v^2 = 15.02 \Rightarrow v = 3.88$ to 3 s.f.

The speed of the box when the string becomes slack is 3.88 m s^{-1} to 3 s.f.

Exercise C (answers p 173)

1 A particle of mass 0.4 kg lies on a smooth horizontal surface attached to one end of an elastic string of natural length 0.8 m and modulus of elasticity 10 N.
The other end of the string is fixed at point A.
The particle is pulled to point B, where $AB = 1.2$ m, and released.

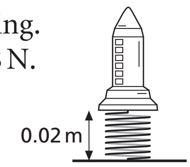


(a) Find the elastic potential energy when the particle is at B.

(b) (i) Use the principle of conservation of energy to find the kinetic energy when the string becomes slack.

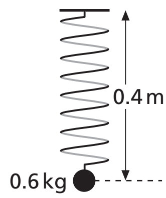
(ii) Hence find the speed of the particle when the string becomes slack.

2 A child's toy rocket, of mass 0.04 kg, is fired by releasing a compressed spring. The natural length of the spring is 0.05 m and its modulus of elasticity is 8 N. The spring is compressed to a length of 0.02 m and released.
Find the maximum height above the surface reached by the rocket.

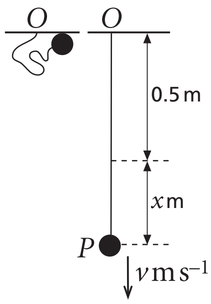


3 A particle of mass 0.6 kg is hung from a spring of natural length 0.2 m and modulus of elasticity 15 N.
The particle is pulled downwards until the spring's length is 0.4 m and is released from rest.

Find the kinetic energy, and hence the speed, of the particle when the string reaches its natural length.



4 A particle of mass 2 kg is attached to one end of a light elastic string of natural length 0.5 m and modulus of elasticity 100 N. The other end of the string is fixed at point O. The particle is held at O and released from rest. When the particle is at point P, the extension of the string is x m and the speed of the particle is $v\,\text{m s}^{-1}$.



(a) Find in terms of x and v an expression for

(i) the gravitational potential energy lost when the particle is at P

(ii) the kinetic energy gained when the particle is at P

(iii) the elastic potential energy gained when the particle is at P

(b) Use the principle of conservation of mechanical energy to show that

$$v^2 = 19.6x - 100x^2 + 9.8$$

(c) Find the maximum value of x.

(d) (i) What is the extension of the string when the kinetic energy is maximum?

(ii) Hence find the maximum speed of the particle.

5 A particle of mass 0.6 kg lies on a rough horizontal surface attached to one end of an elastic string of natural length 0.25 m and modulus of elasticity 20 N.
The other end of the string is fixed to a wall. The coefficient of friction between the particle and the surface is 0.4.
The particle is pulled so that it is 0.5 m from the wall and released from rest.

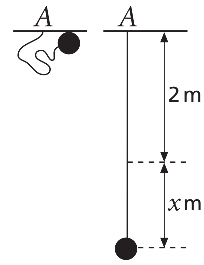
Find the speed of the particle when the string becomes slack.

6 A stuntman of mass 80 kg is attached to one end of a light elastic cord of natural length 7 m and modulus of elasticity 2000 N. The other end of the cord is attached to the top of a building, 15 m above ground level. The stuntman steps off the building and falls vertically.
He can be modelled as a particle throughout the motion.

(a) Show that the speed of the stuntman at the instant the cord first becomes taut is approximately $11.7\,\text{m}\,\text{s}^{-1}$.

(b) The stuntman lands in an airbag positioned 1 m above ground level. Find his speed when he hits the airbag.

7 A particle of mass 10 kg is attached to one end of a light elastic string of natural length 2 m and modulus of elasticity 400 N.
The other end of the string is fixed at point A.
The particle is held at point A and released so that it drops vertically.
When the string has extended x m beyond its natural length, the particle first comes instantaneously to rest.



(a) Show that $50x^2 - 49x - 98 = 0$.

(b) Hence find the extension of the string when the particle comes to rest.

(c) **(i)** Sketch a diagram showing the forces acting on the particle.

(ii) Use Hooke's law to find the tension in the string when the particle comes to rest.

(iii) Use Newton's second law to find the deceleration of the particle at this point.

Key points

- The tension (or thrust), T N, in a spring or elastic string of natural length l m is related to the extension (or compression) x m by the formula

  $$T = \frac{\lambda x}{l} \quad \text{(Hooke's law)}$$

  λ is a constant for a particular spring or elastic string and is known as its modulus of elasticity; it is measured in newtons. (p 114)

- The work done, in joules, in stretching a spring or elastic string with modulus of elasticity λ N and natural length l m by an extension x m is given by

  $$\text{work done} = \frac{\lambda x^2}{2l}$$ (p 118)

- The elastic potential energy (e.p.e.), in joules, of a stretched spring or string is equal to the work done in extending it.

  $$\text{e.p.e.} = \frac{\lambda x^2}{2l}$$ (p 118)

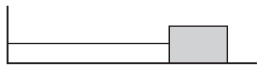
- The total mechanical energy of a system remains constant if no external force other than gravity does work.
  If the principle of conservation of mechanical energy applies, then the sum of the gravitational potential energy, elastic potential energy and kinetic energy of the system is constant. (p 121)

Mixed questions (answers p 173)

1 An elastic string of natural length 0.8 m is stretched to a length of 1.3 m by a force of 16 N.
Find the length of the string when a force of 24 N is applied.

2 A spring has natural length 0.3 m and modulus of elasticity 21 N.
A particle of mass 0.5 kg is attached to one end and hangs in equilibrium.
Find the work done in pulling the particle down a further 0.1 m.

3 A block, of mass 4 kg, is attached to one end of a length of elastic string. The other end of the string is fixed to a wall.



The block is placed on a horizontal surface as shown in the diagram.
The elastic string has natural length 60 cm and modulus of elasticity 60 N.
The block is pulled so that it is 1 metre from the wall and is then released from rest.

(a) Calculate the elastic potential energy when the block is 1 metre from the wall.

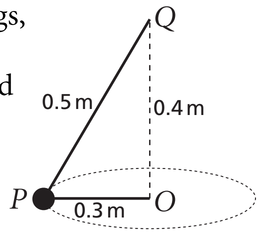
(b) If the surface is smooth, show that the speed of the block when it hits the wall is $2\,\mathrm{m\,s^{-1}}$.

(c) The surface is in fact rough and the coefficient of friction between the block and the surface is 0.3 .

(i) Show that the speed of the block when the string becomes slack is approximately $1.28\,\mathrm{m\,s^{-1}}$.

(ii) Determine whether or not the block will hit the wall. AQA 2002

4 A particle of mass 0.6 kg is attached at the point P to two light strings, QP and OP. Point Q is 0.4 m vertically above O.
String QP is inextensible and of length 0.5 m. String OP is elastic and of natural length 0.25 m and modulus of elasticity λ N.
The particle moves in a horizontal circle, centre O and radius 0.3 m at a constant speed of $3\,\mathrm{m\,s^{-1}}$.



(a) Draw a diagram showing the forces acting on the particle.

(b) Show that the tension in the string QP is 7.35 N.

(c) Find the tension in the string OP in terms of λ.

(d) Find the value of λ.

5 An elastic string has natural length 2 metres and modulus of elasticity λ newtons. One end of the string is fixed at the point O, and a particle of mass 20 kg is attached to the other end of the string.

(a) When in equilibrium the particle is 2.7 metres below O. Show that $\lambda = 560$.

(b) The particle is now held at O and released from rest. The maximum length of the string in the subsequent motion is L.

(i) Show that L satisfies the equation $5L^2 - 27L + 20 = 0$.

(ii) Find the maximum length of the string. AQA 2004

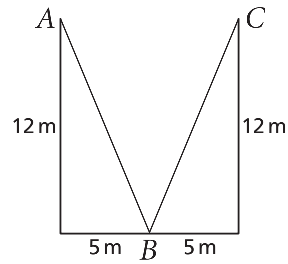
Test yourself (answers p 174)

1 An elastic string of natural length 0.8 m is stretched to a length of 1.3 m by a force of 40 N.
Show that the modulus of elasticity of the string is 64 N.

2 An elastic string has natural length 2 m and modulus of elasticity 40 N.
One end is fixed to a smooth horizontal surface and the other end has a particle of mass 0.8 kg attached.
The string is stretched so that the particle is 2.5 m from the fixed point.
The particle is released from rest and moves along the surface.

Show that the speed of the particle when the string becomes slack is $2.5\,\text{m s}^{-1}$.

3 A 'reverse bungee jump' consists of a 12 metre length of elastic rope, that is stretched into a V-shape *ABC* on a frame, as shown in the diagram. The ends of the elastic rope are fixed to the frame at the points *A* and *C*.



A student, of mass 85 kg, is attached to the midpoint of the elastic rope at *B*. The modulus of elasticity of the elastic rope is 1500 N.

(a) Show that the elastic potential energy of the elastic rope in the initial position shown in the diagram is 12 250 J.

The middle of the rope is then released from *B* and the student moves vertically upwards.

(b) Find the speed of the student, when at a height of 12 metres above *B*.

The student reaches his maximum height before the rope becomes taut again.

(c) Find the maximum height of the student above *B* during the motion. AQA 2001

4 An elastic rope has natural length 4 metres and modulus of elasticity 80 N.
A particle, of mass 2 kg, is attached to one end of the rope, and the other end is fixed at the point *A*. The particle is released from rest at *A* and falls vertically.

(a) When the rope first becomes taut, find

(i) the kinetic energy of the particle

(ii) the speed of the particle

(b) **(i)** The maximum extension of the rope during the motion is x metres.
Show that x satisfies the equation

$$10x^2 - 19.6x - 78.4 = 0$$

(ii) Hence find the maximum length of the rope.

(c) State clearly **one** important assumption that you have made. AQA 2004