Mechanics 1 for AQA contents

4 Momentum

In this chapter you will
- learn what momentum is
- solve problems involving the conservation of momentum

A Mass and momentum (answers p 152)

The **mass** of an object is measured in kilograms. It is the 'quantity of matter' in the object.

Mass is not the same as weight. Weight is the force of gravity acting downwards on an object and is measured in newtons. If the object is moved to the surface of the Moon, the force of gravity there is less than that of the Earth, so the object weighs less on the Moon. But the mass stays the same, wherever the object is.

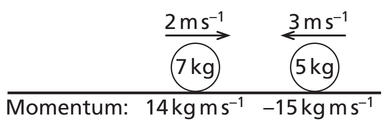
A1 Imagine a light plastic ball and a heavy metal ball.
You kick the light ball and then give a kick of the same strength to the heavy ball.
Which ball will move faster?

The fundamental principles of mechanics were formulated by Isaac Newton (1642–1727). When Newton considered the situation just described, he started with the idea that the same 'amount of kick' should give the same 'amount of motion' to the two objects. He may have thought something like this:

> Imagine that I have a 1 kg object. I give it a kick and it moves away at $10\,\text{m}\,\text{s}^{-1}$. Now imagine that I have a 2 kg object and I give it the same amount of kick. The 2 kg object can be thought of as two 1 kg objects and the kick is shared equally between the two. Each 1 kg object gets half the amount of kick, and so moves at $5\,\text{m}\,\text{s}^{-1}$. In other words, the whole 2 kg object moves at $5\,\text{m}\,\text{s}^{-1}$.

Notice that the product $\text{mass} \times \text{velocity}$ is the same in both cases. Newton called this quantity **momentum**. The kick, when given to a 1 kg object, gives it a momentum of $1 \times 10 = 10\,\text{kg}\,\text{m}\,\text{s}^{-1}$. (Notice the units.) The same kick given to a 2 kg object gives it the same momentum of $2 \times 5 = 10\,\text{kg}\,\text{m}\,\text{s}^{-1}$.

Because velocity is a vector quantity, so is momentum. If objects are all moving along a straight line, momentum is positive in one direction and negative in the other.



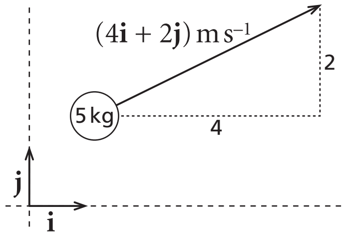
K The momentum of a moving object is the product $\text{mass} \times \text{velocity}$.
Momentum is a vector quantity.

A2 A footballer gives the same kick to two balls, one of mass 0.5 kg and the other of mass 1.5 kg. The first ball moves at $6\,\text{m}\,\text{s}^{-1}$.
What is the velocity of the second?

A3 An object of mass 2.4 kg is kicked and moves with a velocity of $5\,\text{m}\,\text{s}^{-1}$.
The same kick given to a second object causes it to move with a velocity of $3\,\text{m}\,\text{s}^{-1}$.
What is the mass of the second object?

Momentum is a vector quantity. In two dimensions, the momentum of an object may be expressed either as a column vector or in the form $a\mathbf{i} + b\mathbf{j}$.

This diagram shows an object of mass 5 kg moving in two dimensions with a velocity of $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ or $(4\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$.



Its momentum, in $\mathrm{kg\,m\,s^{-1}}$, is mass × velocity

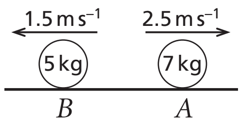
$$= 5(4\mathbf{i} + 2\mathbf{j}) = 20\mathbf{i} + 10\mathbf{j}$$

$$\text{or} \quad 5 \times \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \end{bmatrix}$$

Exercise A (answers p 152)

1 An object of mass 3.5 kg is moving at $4\,\mathrm{m\,s^{-1}}$ on a straight line. Find its momentum.

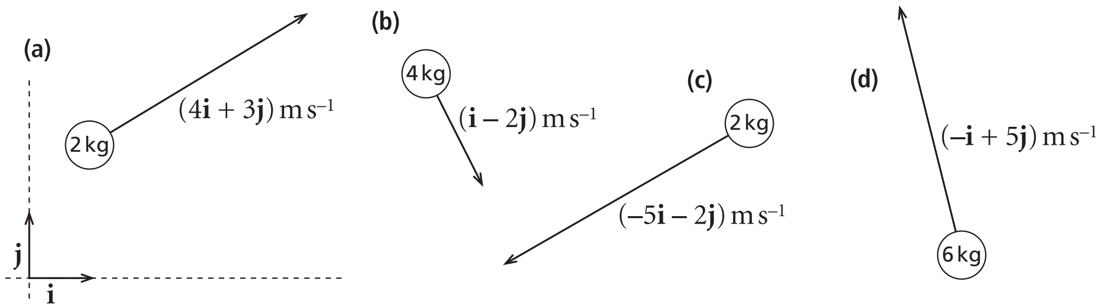
2 Two objects A and B are moving on a straight line.
A has mass 7 kg and is moving forwards at $2.5\,\mathrm{m\,s^{-1}}$.
B has mass 5 kg and is moving backwards at $1.5\,\mathrm{m\,s^{-1}}$.



(a) Find the momentum of A.

(b) Explain why the momentum of B is not $7.5\,\mathrm{kg\,m\,s^{-1}}$ and write down the correct value.

3 Write down the momentum of each of these objects.



4 Write down, as a column vector, the momentum of each of these objects.

(a) An object of mass 5 kg moving with velocity $\begin{bmatrix} 3 \\ 5 \end{bmatrix}\mathrm{m\,s^{-1}}$

(b) An object of mass 3.5 kg moving with velocity $\begin{bmatrix} 6 \\ -4 \end{bmatrix}\mathrm{m\,s^{-1}}$

(c) An object of mass 0.8 kg moving with velocity $\begin{bmatrix} -4 \\ -5 \end{bmatrix}\mathrm{m\,s^{-1}}$

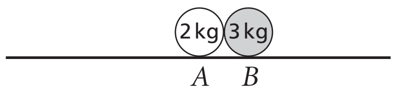
B Conservation of momentum

Imagine two objects A and B moving towards each other on a straight line.

A has a mass of 2 kg and a velocity of $8\,\text{m}\,\text{s}^{-1}$.
B has a mass of 3 kg and a velocity of $-4\,\text{m}\,\text{s}^{-1}$ (that is, $4\,\text{m}\,\text{s}^{-1}$ in the opposite direction).

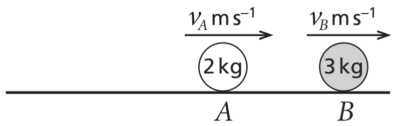
The objects collide with each other.

When they collide, it is as if each object 'kicks' the other. Newton assumed that the two kicks are equal and opposite.



If the two objects are considered together, the total amount of kick on the pair is zero, because the two kicks are equal and opposite.

From this it follows that the total momentum of the two objects will be the same before and after the collision.



Suppose that, after the collision, A moves with velocity v_A and B with velocity v_B.

Before the collision, the momentum of A (in $\text{kg}\,\text{m}\,\text{s}^{-1}$) was $2 \times 8 = 16$, and the momentum of B was $4 \times -3 = -12$.

So the total momentum before the collision was $16 - 12 = 4$.

The total momentum afterwards is $2v_A + 3v_B$.

It follows that $2v_A + 3v_B = 4$.

This equation is not enough to find the values of v_A and v_B. But if one of the values is given, the other can be calculated.

K

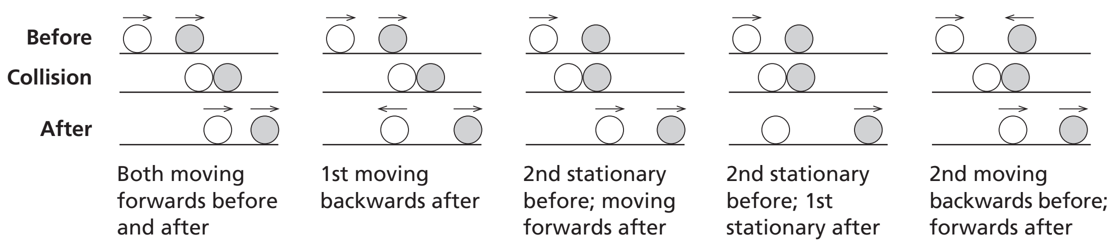
If two objects moving on a straight line collide, the total momentum before the collision is equal to the total momentum after the collision.

Suppose that an object with mass m_1 and velocity u_1 collides with an object with mass m_2 and velocity u_2. Afterwards m_1 moves with velocity v_1 and m_2 with velocity v_2. Then

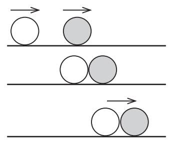
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

This is called the **principle of conservation of linear momentum**.

Here are some of the things that might happen when two objects, moving on a straight line, collide.



There is one case that sometimes arises in problems. This is where the two objects stick together, or **coalesce**, after the collision. They become one combined object.


Objects coalescing after colliding

The mass of the combined object is, of course, the sum of the masses of the individual objects.

Whenever the velocity of an object, before or after the collision, is **backwards**, it must be entered as **negative** in the conservation of momentum equation.

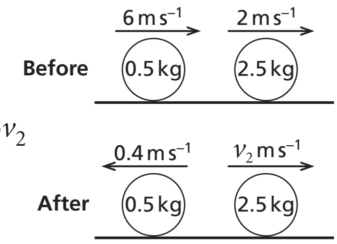
Example 1

An object of mass 0.5 kg moving on a straight line with a speed of 6 m s^{-1} collides with an object of mass 2.5 kg moving in the same direction with a speed of 2 m s^{-1}.

After the collision, the 0.5 kg object moves backwards at 0.4 m s^{-1}.
What are the speed and direction of the other object after the collision?

Solution

Sketch the situation before and after the collision.



Substitute the known values into $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$.

Notice that v_1 *is negative (backwards), so* $v_1 = -0.4$.

From conservation of momentum: $0.5\times6 + 2.5\times2 = 0.5\times-0.4 + 2.5v_2$

$$\Rightarrow \quad 8.2 = 2.5v_2$$

$$\Rightarrow \quad v_2 = 3.28$$

The 2.5 kg object moves at 3.28 m s^{-1} forwards.

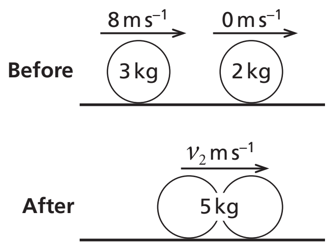
Example 2

An object of mass 3 kg moving on a straight line with a speed of 8 m s^{-1} collides with a stationary object of mass 2 kg.

After the collision the two objects coalesce. Find the speed of the combined object after the collision.

Solution

Sketch the situation before and after the collision.
Notice that the speed of the second object before the collision is 0.
The final speed of the combined object has been called v m s^{-1}.



From conservation of momentum: $3\times8 + 2\times0 = 5v$

$$\Rightarrow \quad 24 = 5v$$

$$\Rightarrow \quad v = 4.8$$

The combined object moves at 4.8 m s^{-1} after the collision.

Exercise B (answers p 152)

1 An object of mass 5 kg moving on a straight line with a speed of $4\,\text{m s}^{-1}$ collides with an object of mass 2 kg moving in the same direction with a speed of $3\,\text{m s}^{-1}$.

Immediately after the collision, the 5 kg object moves forwards at $3.6\,\text{m s}^{-1}$. What are the speed and direction of the other object after the collision?

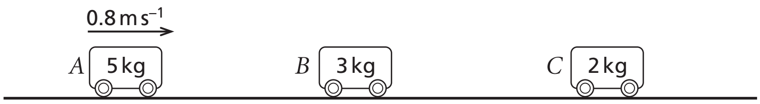
2 An object of mass 3.5 kg moving on a straight line with a speed of $6\,\text{m s}^{-1}$ collides with an object of mass 1.5 kg moving in the opposite direction with a speed of $2\,\text{m s}^{-1}$.

Immediately after the collision, the 3.5 kg object moves forwards at $3\,\text{m s}^{-1}$. What are the speed and direction of the other object after the collision?

3 A model railway truck of mass 0.6 kg is moving along a straight horizontal track with a speed of $0.5\,\text{m s}^{-1}$ when it collides with a stationary truck of mass 0.9 kg.

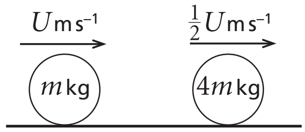
On colliding, the two trucks are coupled and move together at the same speed. Find the speed of the trucks immediately after the collision.

4 Three trucks *A*, *B*, *C*, of masses 5 kg, 3 kg and 2 kg respectively, are on a straight horizontal track as shown in the diagram below. *A* is moving towards *B* with a speed of $0.8\,\text{m s}^{-1}$. *B* and *C* are stationary.



A collides with *B* and attaches itself to *B*. Then the pair collides with *C* and all three trucks are attached together and move with the same speed. Find this speed.

5 A particle of mass m kg is moving in a straight line with a velocity of $U\,\text{m s}^{-1}$. A second particle of mass $4m$ kg is moving in the same straight line, ahead of the first particle, with a velocity of $\frac{1}{2}U\,\text{m s}^{-1}$.



The particles collide and coalesce. Show that the velocity of the combined particle immediately after the collision is $\frac{3}{5}U\,\text{m s}^{-1}$.

6 An object *A* of mass 3 kg, moving in a straight line with a speed of $0.2\,\text{m s}^{-1}$, collides with an object *B*, of mass m kg, moving in the opposite direction with the same speed as *A*.

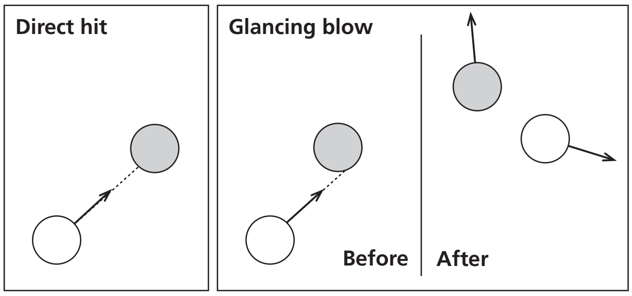
Immediately after the collision *A* is stationary and *B* moves with a speed of $0.3\,\text{m s}^{-1}$. Find the value of m.

7 A particle, *P*, of mass 0.1 kg, is moving in a straight line with speed $3\,\text{m s}^{-1}$ when it collides with a stationary particle, *Q*, of mass 0.5 kg. After the collision, *P* and *Q* move directly away from each other, each with speed $v\,\text{m s}^{-1}$.

Find the value of v.

C Conservation of momentum in two dimensions (answers p 153)

A collision in two dimensions is more complicated than in one dimension.
It is easiest to see this if both objects are thought of as round, like snooker balls.



A snooker ball can give either a direct hit or a 'glancing blow' to another stationary ball. After a direct hit, the second ball moves in the same straight line as the first, so the situation is one-dimensional.

After a glancing blow, the balls move in different directions.

If both balls are moving before they collide, then both may change direction as a result of the collision.

K The principle of conservation of momentum applies to motion in two (or three) dimensions. In vector form, the equation is

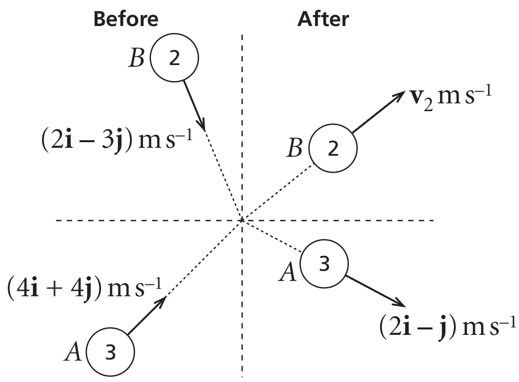
$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

When applying conservation of momentum, you do not need to worry about exactly what happens when the objects collide. Only the situations before and after the collision are relevant.

C1 An object A of mass 3 kg has a velocity of $(4\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$.

It collides with an object B of mass 2 kg travelling with velocity $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$.

Afterwards, A moves with velocity $(2\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$ and B with velocity $\mathbf{v}_2\,\text{m s}^{-1}$ (unknown).



If the known quantities are substituted in the conservation of momentum equation you get

$$3(4\mathbf{i} + 4\mathbf{j}) + 2(2\mathbf{i} - 3\mathbf{j}) = 3(2\mathbf{i} - \mathbf{j}) + 2\mathbf{v}_2$$

By solving this equation, show that $\mathbf{v}_2 = 5\mathbf{i} + 4.5\mathbf{j}$.

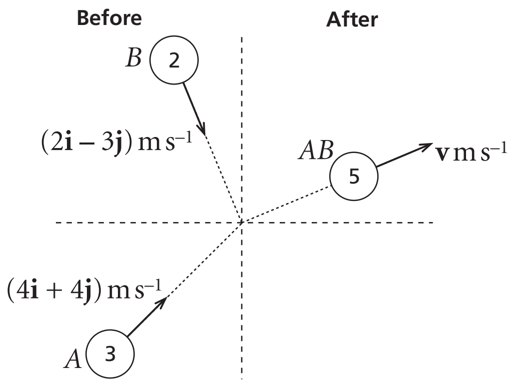
Question C1 can also be done using column vector notation. If the unknown final velocity is called $\begin{bmatrix} a \\ b \end{bmatrix}$, the equation becomes $3\begin{bmatrix} 4 \\ 4 \end{bmatrix} + 2\begin{bmatrix} 2 \\ -3 \end{bmatrix} = 3\begin{bmatrix} 2 \\ -1 \end{bmatrix} + 2\begin{bmatrix} a \\ b \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 16 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 + 2a \\ -3 + 2b \end{bmatrix} \Rightarrow a = 5,\ b = 4.5$$

C2 Suppose the situation before the collision is the same as in C1, but that the two objects coalesce and move together with velocity $\mathbf{v}$.

Write down the equation of conservation of momentum in this case.

Solve the equation to find the value of $\mathbf{v}$.



Example 3

An object of mass 8 kg, moving with velocity $(3\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$, collides with a stationary object of mass 2 kg.

Immediately after the collision, the velocity of the 2 kg object is $(\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$. Find the velocity after the collision of the 8 kg object.

Solution

As usual, sketch the situation before and after the collision.

From the conservation of momentum:

$$8(3\mathbf{i} + 2\mathbf{j}) = 8\mathbf{v} + 2(\mathbf{i} + \mathbf{j})$$

$$\Rightarrow \quad 24\mathbf{i} + 16\mathbf{j} = 8\mathbf{v} + 2\mathbf{i} + 2\mathbf{j}$$

$$\Rightarrow \quad 22\mathbf{i} + 14\mathbf{j} = 8\mathbf{v}$$

$$\Rightarrow \quad \mathbf{v} = 2.75\mathbf{i} + 1.75\mathbf{j}$$

The velocity of the 8 kg object after the collision is $(2.75\mathbf{i} + 1.75\mathbf{j})\,\text{m s}^{-1}$.

Exercise C (answers p 153)

1 An object of mass 3 kg, moving with velocity $(4\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$, collides with an object of mass 2 kg, moving with a velocity of $(2\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$.

Immediately after the collision, the velocity of the 3 kg object is $(3\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$. Find the velocity after the collision of the 2 kg object.

2 An object of mass 4 kg, moving with velocity $\begin{bmatrix} -1 \\ 3 \end{bmatrix}\text{m s}^{-1}$, collides with an object of mass 3 kg, moving with velocity $\begin{bmatrix} 2 \\ -1 \end{bmatrix}\text{m s}^{-1}$.

Immediately after the collision, the 4 kg object moves with velocity $\begin{bmatrix} 2 \\ 0 \end{bmatrix}\text{m s}^{-1}$. Find the velocity after the collision of the 3 kg object.

3 A particle of mass 0.35 kg travelling with velocity $(3\mathbf{i} - 5\mathbf{j})\,\text{m s}^{-1}$ collides with a particle of mass 0.15 kg travelling with velocity $(\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$. The two particles coalesce. Find the velocity of the pair of particles immediately after the collision.

4 A particle P of mass 0.03 kg, travelling with velocity $\mathbf{u}\,\mathrm{m\,s^{-1}}$, collides with a stationary particle Q of mass 0.01 kg. Immediately after the collision, P moves with velocity $\begin{bmatrix} 4 \\ -1 \end{bmatrix}\mathrm{m\,s^{-1}}$ and Q with velocity $\begin{bmatrix} 3 \\ 3 \end{bmatrix}\mathrm{m\,s^{-1}}$. Find the value of $\mathbf{u}$.

5 A moving particle A collides with another moving particle B.
The mass of A is 1.5 kg and its velocities before and after the collision are $(4\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$ and $(2\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-1}}$ respectively.

(a) (i) Find the momentum lost by A as a result of the collision, giving the units.

(ii) State the momentum gained by B as a result of the collision.

(b) The mass of B is 0.5 kg. Immediately after the collision the velocity of B is $(\mathbf{i} + \mathbf{j})\,\mathrm{m\,s^{-1}}$. Find the velocity of B immediately before the collision.

Key points

- The momentum of an object is defined as mass × velocity.
  Momentum is a vector quantity. Its units are $\mathrm{kg\,m\,s^{-1}}$. (p 73)

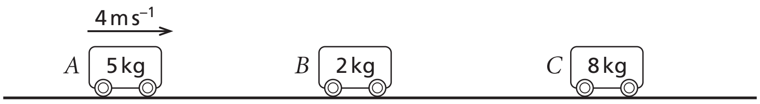
- If an object of mass m_1 travelling with velocity $\mathbf{u}_1$ collides with an object of mass m_2 travelling with velocity $\mathbf{u}_2$, and if $\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocities of the objects immediately after the collision, then

  $$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

  This is called the principle of conservation of momentum. (pp 75, 78)

Mixed questions (answers p 153)

1 Three trucks A, B and C, of masses 5 kg, 2 kg and 8 kg respectively, are situated on a smooth horizontal track as shown in the diagram. Initially A is moving at $4\,\mathrm{m\,s^{-1}}$ towards B, and B and C are stationary.



(a) After A collides with B, A's speed is reduced to $2\,\mathrm{m\,s^{-1}}$.
Find the speed of B after this collision.

(b) B then collides with C. B and C are coupled together in the collision.
Find the speed of the pair B, C after this collision.

(c) Finally A collides with the pair B, C and all three trucks are coupled together.
Find the final speed of the three trucks.

2 A particle A of mass m kg, moving in a straight line with speed $u\,\mathrm{m\,s^{-1}}$, collides with a second particle B, of mass $4m$ kg, moving in the opposite direction with speed $u\,\mathrm{m\,s^{-1}}$.
As a result of the collision, the direction of A is reversed and A moves with speed $\frac{1}{2}u\,\mathrm{m\,s^{-1}}$.
Show that the speed of B after the collision is $\frac{5}{8}u\,\mathrm{m\,s^{-1}}$.

3 A particle of mass 0.45 kg, moving with velocity $(-2\mathbf{i} + 4\mathbf{j})\ \text{m s}^{-1}$, collides with a particle of mass 0.35 kg, moving with velocity $(2\mathbf{i} - 6\mathbf{j})\ \text{m s}^{-1}$. In the collision the particles coalesce. Find, to 3 s.f., the **speed** of the particles after the collision.

4 A particle P has mass 5 kg. It is moving along a straight line with speed $4\ \text{m s}^{-1}$, when it collides directly with another particle Q which is at rest. The mass of Q is m kg.

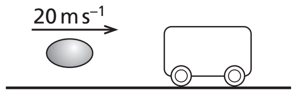
After the collision P moves with a speed of $1.2\ \text{m s}^{-1}$ and Q moves with a speed of $1.4\ \text{m s}^{-1}$.

(a) If P and Q both move in the same direction after the collision, show that $m = 10$.

(b) If P and Q move in opposite directions after the collision, find m. AQA 2002

Test yourself (answers p 153)

1 A trolley, of mass 10 kg, is placed at rest on a set of straight horizontal rails. Large pellets, each of mass 0.5 kg, are fired at the trolley. When each pellet hits the trolley, the pellet is travelling horizontally and parallel to the rails at a speed of $20\ \text{m s}^{-1}$. When the pellets hit the trolley, they stick to it and continue to move with the trolley. Assume that there is no resistance to the motion of the trolley.



(a) Show that the speed of the trolley after it has been hit by the first pellet is $\frac{20}{21}\ \text{m s}^{-1}$.

(b) Find the speed of the trolley after it has been hit by the second pellet. AQA 2003

2 A stone, A, of mass 2 kg, is sliding across a smooth horizontal surface with velocity $\begin{bmatrix} 3 \\ 1 \end{bmatrix}\ \text{m s}^{-1}$. It hits another stone, B, of mass 3 kg sliding across the same surface with velocity $\begin{bmatrix} -4 \\ -3 \end{bmatrix}\ \text{m s}^{-1}$.

Stone A rebounds with velocity $\begin{bmatrix} -3 \\ -2 \end{bmatrix}\ \text{m s}^{-1}$. Find the velocity of B after the collision.

3 A moving particle P collides with a particle Q, which is also moving. The particle P is of mass 0.25 kg, and its velocities immediately before and after the collision are $\begin{bmatrix} 3 \\ 6 \end{bmatrix}\ \text{m s}^{-1}$ and $\begin{bmatrix} -1 \\ 4 \end{bmatrix}\ \text{m s}^{-1}$ respectively.

(a) (i) Find the momentum lost by P due to the collision. State the units of your answer.

(ii) State the momentum gained by Q due to the collision.

(b) The mass of Q is 0.1 kg.
Immediately after the collision, Q moves with velocity $\begin{bmatrix} 5 \\ 2 \end{bmatrix}\ \text{m s}^{-1}$.

Find the velocity of Q immediately before the collision. AQA 2003