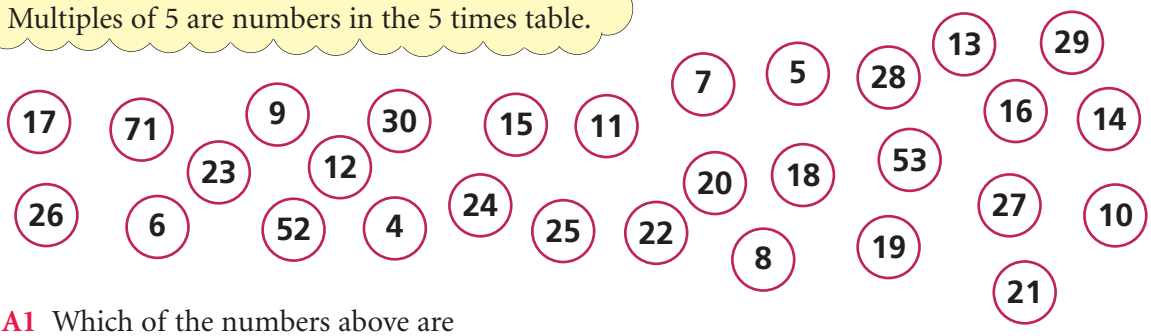
17 Number relationships

This work will help you

- revise multiples, factors and square numbers.
- find square roots

A Multiples

Multiples of 4 are numbers in the 4 times table.
Multiples of 5 are numbers in the 5 times table.



A1 Which of the numbers above are

(a) multiples of 4 (b) multiples of 3

(c) multiples of 5 (d) 1 more than a multiple of 5

(e) 2 more than a multiple of 3

A2 Here is a way of telling whether a number is a multiple of 3.

Is 1527 a multiple of 3?

Add together the digits 1 + 5 + 2 + 7 = 15.

Since the sum, 15, is a multiple of 3 then so is 1527.

Use this method to say which of these numbers are multiples of 3.

165 237 371 2652 3433 7632 26 571

A3 Sharon has some sweets.
She could share them equally between 3 people and have none left.
Or she could share them equally between 4 people and have none left.
Or she could share them equally between 5 people and still have none left.

How many sweets has Sharon got?

A4 1■1 is a multiple of 9. What digit does ■ stand for?

A5 All these numbers are multiples of 9.
What digit does each ■ stand for?

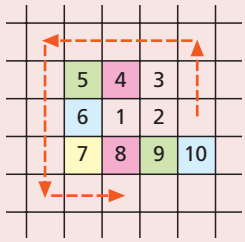
(a) 2■ (b) 5■ (c) 8■ (d) ■6 (e) 15■

(f) ■07 (g) ■04 (h) ■00 (i) 1■00

Patchwork patterns

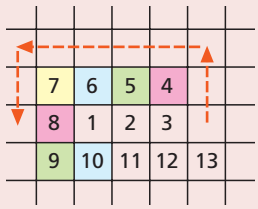
Take some squared paper and number the squares in a spiral.
Go up to 100 at least.

- Shade the multiples of 4 in one colour.
- Use another colour for numbers which are
  1 more than a multiple of 4,
  a third colour when they are 2 more,
  and a fourth colour when they are 3 more.



What happens if you use a different number (not 4)?

What happens with a different spiral?



B Factors

24 can be divided exactly by 4. We say 4 is a **factor** of 24.

Writing down all the possible multiplication pairs which give 24 uses all the factors of 24.

$1 \times 24 = 24$ $2 \times 12 = 24$ $3 \times 8 = 24$ $4 \times 6 = 24$

So the factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

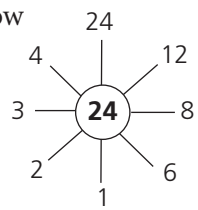
B1 Which of these numbers are factors of 20?

4 5 6 2 40 8 10 1 3 20

B2 Write down all the multiplication pairs for each of these numbers.
Use the pairs to write down all the factors of each number.

(a) 21 (b) 18 (c) 48 (d) 30 (e) 16

B3 A **factor spider** can be used to show all the factors of a number.



This is a factor spider for 24.

Draw factor spiders for these numbers.

(a) 20 (b) 18 (c) 36 (d) 42 (e) 23

B4

W	A	N	D	S		M	I	G	H	T		H	A	V	E		C	A	U	G	H	T		I	N		S	E	A	W	E	E	D	?
28	13	6	40	56		8	26	21	7	49		16	34	10	27		17	52	4	9	88	38		47	2		23	95	37	46	14	18	11		

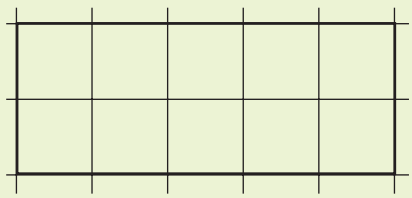
Copy the sentence and the numbers above on squared paper.
Cross out any letter which has below it a number which is

- a multiple of 3
- a multiple of 5
- a factor of 14
- a factor of 32
- a number with 11 as a factor

Write the sentence you are left with.

T

Multiples and factors bingo



Draw a bingo card with room for ten numbers (or use squared paper).

On your card write any ten numbers from this list (no repeats):

4 6 7 10 12 15 16 18 20 21 24 27 29 30 31 33 35 36 39 40

Your teacher will tell you which numbers you can cross out using multiples and factors.

C Prime numbers

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The sieve of Eratosthenes

You need some squared paper.

Write the numbers 1, 2, 3, 4, 5, … across your paper.

Write another 2 under the 2 in the top line.
Draw a line all the way across the page underneath this.
This line is the 'sieve'.
Write any other multiples of 2 below the line in the correct squares.
Carry on to the edge of the page.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	2														
			4		6		8		10		12		14		16

The next number after 2 which hasn't yet been written is 3.
Write 3 above the line.
Now write the other multiples of 3 below the line in the correct squares.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	2	**3**													
			4		**6**		8	**9**	10		**12**		14	**15**	16

The next new number that hasn't yet been written is 5, so write 5 above the line and other multiples below. Carry on like this until you can go no further.

Which numbers have not fallen through the sieve?

The numbers that have not fallen through the sieve are called **prime numbers**.
Prime numbers have no factors, other than 1 and themselves.

C1 Use your sieve to check which of these are prime numbers.

37 19 27 51 33

C2 Work out which of these numbers are prime numbers.

95 83 121 99 345

D Squares and square roots

When you multiply a number by itself you get a **square number**.

25 is a square number because $5 \times 5 = 25$. We write $5^2 = 25$.

Happy numbers

23 is 'happy' because

$$23 \rightarrow 2^2 + 3^2 = 4 + 9 = 13 \rightarrow 1^2 + 3^2 = 1 + 9 = 10 \rightarrow 1^2 + 0^2 = 1 + 0 = 1$$

If the number you end up with is 1, the number you started with is happy.

If you reach another single-digit number, the number you started with is sad.

$$42 \rightarrow 4^2 + 2^2 = 16 + 4 = 20 \rightarrow 2^2 + 0^2 = 4 + 0 = 4$$

- Which numbers less than 50 are happy?

D1 Which of these are square numbers?

36 50 49 100 99

D2 Copy the sentence and the numbers below.
Cross out any letter which has below it a square number.

D	O	G	S		N	O	T	I	C	E		C	H	I	L	D	R	E	N		L	O	O	K	I	N	G		A	T		T	R	E	A	S	U	R	E
35	10	25	49		12	60	8	9	16	81		4	26	45	36	70	64	24	1		100	36	16	4	40	15	9		50	25		44	56	32	36	81	16	49	72	

Write the sentence you are left with.

D3 Copy this equation.

$\square + \square + \boxed{6^2} + \boxed{7^2} = \square + \square + \boxed{5^2} + \boxed{8^2}$

Fit these numbers into the empty boxes to make the equation true. 1^2 2^2 3^2 4^2

D4 What number is squared to make

(a) 9 (b) 64 (c) 25 (d) 49

4 is squared to make 16. 4 is called the **square root** of 16. We write $\sqrt{16} = 4$

D5 What is the square root of

(a) 36 (b) 4 (c) 81 (d) 100

D6 What are

(a) $\sqrt{64}$ (b) $\sqrt{1}$ (c) $\sqrt{9}$ (d) $\sqrt{49}$

D7 Use the √ key on your calculator to find

(a) $\sqrt{196}$ (b) $\sqrt{625}$ (c) $\sqrt{361}$ (d) $\sqrt{529}$

(e) $\sqrt{841}$ (f) $\sqrt{749}$ (g) $\sqrt{202}$ (h) $\sqrt{264}$

What progress have you made?

Statement	Evidence
I can find multiples.	**1** Write four multiples of 6.
	2 Which of these numbers are multiples of 8? 40 28 36 72 76 56
I can find factors.	**3** Write all the factors of (a) 48 (b) 30 (c) 36 (d) 100
I can spot square numbers.	**4** Which of these numbers are square numbers? 24 64 36 54 20 25
I can find square roots.	**5** What is (a) $\sqrt{81}$ (b) $\sqrt{400}$ (c) $\sqrt{121}$ (d) $\sqrt{345}$

17 Number relationships

This unit provides revision on multiples, factors and square numbers. It includes several investigations and puzzles. Square roots are introduced as the inverse of squares.

Number bites W1 and W4 to W6 on pages 4 and 5 offer suitable revision for weaker pupils before they start this unit.

Essential	**Optional**
Squared paper	Coloured pens or pencils
Practice booklet pages 41 and 42	

A Multiples (p 94)

Squared paper
Optional: Coloured pens or pencils

A2 Pupils could also look at other ways of deciding whether a large number is a multiple of another number, such as:

A number is a multiple of …

6 if it is a multiple of 3 and even
4 if the last two digits are divisible by 4
8 if half of it is divisible by 4
9 if the sum of its digits is divisible by 9

A5 Pupils could make up their own puzzles like these. If they give them to others to do they will probably discover that there are not always unique answers. This can be discussed.

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Patchwork patterns

◊ This activity is designed to allow practice in finding multiples rather than to form the basis of an in-depth investigation. More able pupils might consider how the pattern is affected by the way the spiral is formed.

𝔹 Factors (p 95)

In finding all the factors of a number pupils should be encouraged to use a systematic approach using multiplication pairs. They should list the factors in numerical order.

B4 You may wish to draw attention to the fact that the sentence remaining (Was it a cat I saw?) is palindromic.

Multiples and factors bingo

T

You will need to call out statements such as 'A multiple of 7' or 'A factor of 36'.

Pupils could be asked to choose their own numbers between 1 and 40 and after playing discuss what numbers were good ones to pick.

ℂ Prime numbers (p 96)

Squared paper

The sieve of Eratosthenes

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◊ The prime numbers will not fall through the sieve – encourage pupils to describe these numbers as clearly as they can in their own words. The primes that it reveals will depend on the number of squares on the width of the paper.

Eratosthenes of Cyrene (276–194 BC) was a Greek poet, astronomer, mathematician, historian and athlete. He was librarian of the University of Alexandria. Apart from the 'sieve', he is well known for measuring the circumference of the Earth by observing the direction of the Sun at two places a great distance apart.

𝔻 Squares and square roots (p 97)

Happy numbers

T

◊ This can be organised very successfully as a class activity, with different pupils investigating different numbers. The results can then be combined in one big diagram. A chart can be produced showing how many stages it takes for each starting number.

The happy numbers between 0 and 50 are

1, 7, 10, 13, 19, 23, 28, 31, 32, 44, 49

𝔸 Multiples (p 94)

A1 (a) 4, 8, 12, 16, 20, 24, 28, 52
(b) 6, 9, 12, 15, 18, 21, 24, 27, 30
(c) 5, 10, 15, 20, 25, 30
(d) 6, 11, 16, 21, 26, 71
(e) 5, 8, 11, 14, 17, 20, 23, 26, 29, 53, 71

A2 165, 237, 2652, 7632, 26 571

A3 Any multiple of 60

A4 1**7**1

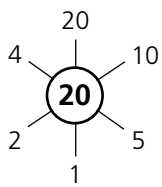
A5 (a) 2**7** (b) 5**4** (c) 8**1**
(d) **3**6 (e) 15**3** (f) **2**07
(g) **5**04 (h) **9**00 (i) 1**8**00

𝔹 Factors (p 95)

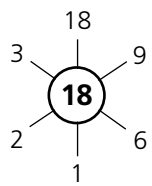
B1 1, 2, 4, 5, 10, 20

B2 (a) $1 \times 21 = 21$, $3 \times 7 = 21$,
so factors are 1, 3, 7 and 21
(b) $1 \times 18 = 18$, $2 \times 9 = 18$, $3 \times 6 = 18$,
so factors are 1, 2, 3, 6, 9 and 18
(c) $1 \times 48 = 48$, $2 \times 24 = 48$, $3 \times 16 = 48$, $4 \times 12 = 48$, $6 \times 8 = 48$,
so factors are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
(d) $1 \times 30 = 30$, $2 \times 15 = 30$, $3 \times 10 = 30$, $5 \times 6 = 30$,
so factors are 1, 2, 3, 5, 6, 10, 15, 30
(e) $1 \times 16 = 16$, $2 \times 8 = 16$, $4 \times 4 = 16$,
so factors are 1, 2, 4, 8, 16

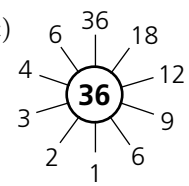
B3 (a)



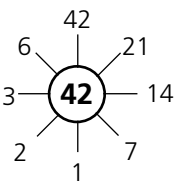
(b)



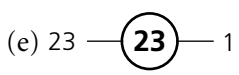
(c)



(d)



(e)



B4 Was it a cat I saw?

ℂ Prime numbers (p 96)

C1 37 and 19 are prime.
27, 51 and 33 are not.

C2 Only 83 is prime.

𝔻 Squares and square roots (p 97)

D1 36, 49 and 100

D2 Do not hide in a tree.

D3 $\mathbf{1^2} + \mathbf{4^2} + 6^2 + 7^2 = \mathbf{2^2} + \mathbf{3^2} + 5^2 + 8^2$

D4 (a) 3 (b) 8 (c) 5 (d) 7

D5 (a) 6 (b) 2 (c) 9 (d) 10

D6 (a) 8 (b) 1 (c) 3 (d) 7

D7 (a) 14 (b) 25 (c) 19
(d) 23 (e) 29 (f) 27.36…
(g) 14.21… (h) 16.24…

What progress have you made? (p 98)

1 The pupil's four multiples of 6

2 40, 72, 56

3 (a) 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
(b) 1, 2, 3, 5, 6, 10, 15, 30
(c) 1, 2, 3, 4, 6, 9, 12, 18, 36
(d) 1, 2, 4, 5, 10, 20, 25, 50, 100

4 64, 36 and 25

5 (a) 9 (b) 20 (c) 11 (d) 18.57…

Practice booklet

Sections A and B (p 41)

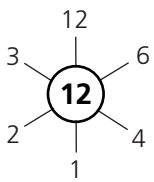
1 (a) 20, 30 (b) 20, 25, 30, 35
(c) 12, 16, 20, 36 (d) 12, 36

2 (a) 23**1** or 23**4** or 23**7**
(b) 23**2** or 23**6**
(c) 23**4**

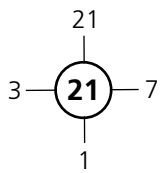
3 (a) No (b) Teddy
(c) No (d) Lollipop
(e) Teddy
(f) Teddy and lollipop

4 (a) 1×15, 3×5
so factors of 15 are 1, 3, 5, 15
(b) 1×27, 3×9
so factors of 27 are 1, 3, 9, 27
(c) 1×32, 2×16, 4×8
so factors of 32 are 1, 2, 4, 8, 16, 32
(d) 1×36, 2×18, 3×12, 4×9, 6×6
so factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36

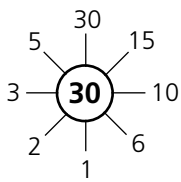
5 (a)



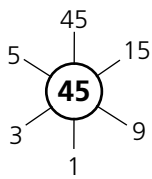
(b)



(c)



(d)



6 5

Sections C and D (p 42)

1 11, 13, 17, 19, 23, 29

2 97, 109, 137

3 (a) 1 (b) 25 (c) 81 (d) 121

4 Yes, 8 slabs by 8 slabs

5 (a) 2 (b) 4 (c) 6 (d) 7

6 (a) 7 (b) 9

7 (a) 13 (b) 17 (c) 22 (d) 31